Sequential Bidding in Day-Ahead Auctions for Spot Energy and Power Systems Reserve

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Abstract

In this paper a novel approach for sequential bidding on day-ahead auction markets for spot energy and power systems reserve is presented. For the spot market a relatively simple method is considered as a competitive market is assumed. For the reserve market one bidder is assumed to behave strategically and the behavior of the competitors is summarized in a probability distribution of the market price. This results in a method for sequential bidding, where the bidding prices and capacities on the spot and reserve markets are calculated by maximizing a stochastic non-linear objective function of expected profit. With an exemplary application is shown that the trading sequence leads to increasing bidding capacities and prices in the reverse rank number of the markets. Hence, the consideration of a defined trading sequence greatly influences the mathematical representation of the optimal bidding behavior under price uncertainty in day-ahead auctions for spot energy and power systems reserve.

Key words: Bayes-strategy, Decision support, Multi-unit, Pay-as-bid, Power systems reserve, Price uncertainty, Procurement auction, Sequential bidding.

1 Introduction

In many liberalized electricity markets a new market segment has recently been established: competitive tendering of power systems reserves (for the German case cf. Swider and Weber, 2003). Thereby all generation units fulfilling defined

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requirements are allowed to bid in these markets. This development leads potential bidders to decide whether to bid an available capacity on the already established spot or on these recently commenced reserve markets.

In order to develop appropriate methods for bidding on these markets the respective market design is of special importance. Within this paper and following the operation of the auction markets in Germany the spot market is considered to be a uniform-priced double-sided call auction. The reserve markets are considered to be pay-as-bid one-sided procurement auctions.

Decision support for bidding on uniform-priced auction markets for spot electricity has been studied with extended approaches of optimal unit-commitment (cf. e. g. Kagiannas et al., 2004; Yamin, 2004, and the references therein). By analysing these approaches may be seen that the liberalization of electricity markets was followed by bidding models focussing on profit maximization. Thereby deterministic and stochastic approaches may be distinguished.

Deterministic approaches have been presented e. g. by Conejo et al. (2002) and Rodriguez and Anders (2004). Within these papers the *ex ante* unknown market price is considered to follow a stochastic price process described with econometric models. In both papers only the expected value of the price is used, not the price distribution. However, Rodriguez and Anders (2004) propose to cover the existing uncertainties of the market price by considering several defined scenarios of the market price. But these scenarios are evaluated separately resulting in a number of separate sub-optimal bidding solutions.

An improved consideration of the uncertain market price is possible by applying methods of stochastic programming (cf. Birge and Louveaux, 1997). Such approaches have been proposed for profit maximized bidding on electricity spot markets e. g. by Takriti et al. (2000) and Ni et al. (2004). Within these papers a discrete subset of the uncertain market price is considered. The different scenarios are evaluated simultaneously resulting in just one optimal bidding solution. By applying such an approach one problematic aspect is to guarantee that the considered subset of the stochastic market price sufficiently represents the continuous price distribution. To overcome this problem the continuous probability density function may be considered directly.

Friedman (1956) proposed to calculate a probability distribution $F^{\mathrm{C}}(p)$ for the best competitive bid p^{C} and then had the bidder choose the bid p^{B} that maximized her expected profit. With the probability of accepting this bid $\mathrm{P}^{\mathrm{A}}(p^{\mathrm{C}} > p^{\mathrm{B}})$, that is equal to the probability of p^{C} being higher than p^{B}, the expected profit is given by $(p^{\mathrm{E}} - p^{\mathrm{B}})\mathrm{P}^{\mathrm{A}}(p^{\mathrm{C}} > p^{\mathrm{B}})$. Thereby p^{E} indicates a known expected value for the auctioned asset. $F^{\mathrm{C}}(p)$ was proposed to be calculated as the distribution of the maximum of independent draws from the bid distributions of each competitor. Hence, for calculating the probability of acceptance, $P^{A}(p^{C} > p^{B}) = 1 - F^{C}(p^{B})$, the bids of all rival bidders and even their number need to be known.

The main problem with this approach is the availability of detailed price data on historic bids of the rival bidders. A much better approach could be to summarize the behavior of the competing bidders in a joint probability distribution of the market price $F^{M}(p)$. This results in a probability of accepting one's bidder's offer given by $P^{A}(p^{M} > p^{B}) = 1 - F^{M}(p^{B})$ with the bidding price p^{B} and the market price p^{M} (seen as a stochastic variable). The approach was first proposed for general single-unit uniform-priced auctions by Hanssmann and Rivett (1959) and later by Lavalle (1967). An extension of this approach specifically addressing bidding on electricity spot markets designed as multi-unit uniform-priced double auctions has recently been presented by Anderson and Philpott (2002). Thereby the authors study the profit maximizing offer stack of a bidder by considering price and demand uncertainty reflected by what they call a market distribution function, i. e. a joint probability distribution of market price and demand. In case of a multi-unit pay-as-bid procurement auction, however, one has to take into account that no single probability distribution of the market price is readily available (in this paper and following today's markets for power systems reserve in Germany demand is assumed to be known) and that a non-negligible bid of the considered bidder may have considerable influence on the relevant market price in subsequent periods. Solutions to these problems have recently been presented by Swider and Weber (2005) and will briefly be discussed in the following.

Simultaneous bidding on spot and reserve markets has been studied by Wen and David (2002). In difference to the design of the reserve markets to be considered here a uniform-priced auction is considered and available probability distributions of the bidding prices for each rival bidder are assumed (cf. the discussed approach by Friedman, 1956). Following the methodology proposed by Swider and Weber (2005) simultaneous bidding on spot and reserve markets with a pay-as-bid auction design of the considered reserve markets has recently been discussed by Swider (2005a). However, the literature still lacks methods for sequential bidding in electricity auctions (cf. Rothkopf, 1999, for a discussion on the need of bidding models able to consider the daily repetition in electricity auctions). In this paper the bidding approaches discussed in Swider and Weber (2005) and Swider (2005a) are extended to cover sequential bidding in day-ahead auctions of spot energy and power systems reserve.

Sequential bidding may generally be considered by taking the aforementioned probability of accepting one's bidder's offer in an earlier auction into account. With the bidding price p^{B}, the stochastic variable of the market price p^{M} and the probability distribution of the market price $F^{M}(p)$ the probability of acceptance is given by $P^{A}(p^{M} > p^{B}) = 1 - F^{M}(p^{B})$. Hence, if a bidder bids the capacity L_{1}^{B} with price p_{1}^{B} in an earlier auction, she expects the bid to be accepted with the probability $\mathrm{P}^{\mathrm{A}}(p_1^{\mathrm{M}} > p_1^{\mathrm{B}})$. In a subsequent auction she may then consider to offer the capacity $L_2^{\mathrm{B}} = L_1^{\mathrm{B}}$ again, though with price p_2^{B} and reduced by weighting the capacity with the probability of rejection in the earlier auction. In the subsequent auction then follows $\tilde{L}_2^{\mathrm{B}} = \mathrm{P}^{\mathrm{R}}(p_1^{\mathrm{M}} \leq p_1^{\mathrm{B}})\, L_1^{\mathrm{B}}$ for the expected bidding capacity. Generally the probability of rejection is given by $\mathrm{P}^{\mathrm{R}}(p^{\mathrm{M}} \leq p^{\mathrm{B}}) = F^{\mathrm{M}}(p)$. This fundamental approach has originally been developed for repeated auctions and has intensively been discussed in the literature (cf. e. g. Kortanek et al., 1973; Oren and Rothkopf, 1975, and many others). This general approach and its application to sequential bidding in day-ahead auctions of spot energy and power systems reserve will be discussed in more detail in the following.

In principle, sequential bidding leads to consider that the value of the capacity offered in an earlier auction increases. Reiß and Schöndube (2003) consider two sequential single-unit single-part procurement auctions in a game theoretic analysis and show that a bidder will bid with higher prices in the earlier auction. Hence, the bidder is aware of (profitable) subsequent bidding opportunities and reduces the probability of acceptance in the earlier auction. Comparable results have also been presented by Kittsteiner et al. (2004). The authors consider several sequential first- and second-price-sealed-bid auctions also in a game theoretic analysis and show that the trading sequence leads to decreasing bidding prices in the rank number of the markets. It may *a priori* be expected that these results can also be seen if sequential bidding in electricity auctions is considered.

The paper is organized as follows: In Section 2 the considered electricity day-ahead spot and reserve markets are described. The methodologies for sequential bidding on such markets are presented in Section 3. In Section 4 the results of an exemplary estimation exercise are discussed. Finally, in Section 5 conclusions and indications for further research are given.

2 Day-Ahead Auction Markets

Following the deregulation of the electricity sector in Germany several auction markets have been established: one spot and four reserve markets.

2.1 Spot Market

The spot market in Germany is operated by the European Energy Exchange AG (EEX) and commenced trading for physical contracts in June 2000. Trading is executed day-ahead in a double-sided call auction. Thereby participants submit bids for purchase and sale of hourly contracts for the following day. The bids are collected in a closed order book. Every trading day at noon the individual supply and demand curves are aggregated to a single supply and demand curve. The intersection between the two curves represents the balance between purchase and sale bids and determines the uniform market-clearing price. As long as there are no transmission constraints the spot market price is the same for all Germany. On average, about 5400 MW were traded each hour on the German spot market in 2003 (the total trading volume was about 9.6 % of the electricity consumption).

2.2 Reserve Market

As electricity cannot be stored in any major quantities, the amounts of electricity generated and consumed have to match exactly. Within a defined region this system balancing is in the responsibility of a transmission system operator (TSO). The TSO must guarantee to have enough excess generation available for use at all times so that if e. g. one generator fails, all loads may still be served without interruption. The quantity of this power systems reserve is defined *ex ante* and in the system of the Union for the Co-ordination of Transmission of Electricity (UCTE) three qualities are differentiated. They differ in terms of the activation and response speed. Primary and secondary reserve are automatically called while tertiary reserve is called via rescheduling of generation. Primary reserve must be fully provided within 30 seconds, secondary reserve within 5 minutes and tertiary reserve within 15 minutes.

Previous to the liberalization of the electricity market the four TSO in Germany predominantly procured the reserve capacities within the same company. In 2001 the market opened due to requirements during merger control. Today the TSO run auction markets to procure power systems reserve by way of competitive tendering. These procurement auctions are characterized by simultaneous tendering of multiple generation units. Any bid consists of the offered capacity and two prices. One price is for holding the capacity in reserve (capacity price) and the other for delivery in case of actual use (energy price). For the remuneration of the accepted bids the pay-as-bid method is applied. Given the different reserve qualities this paper focuses on incremental tertiary only, as it is traded day-ahead (incremental reserve is procured to balance a shortage of supply). The proposed approaches may, however, be easily applied to consider other incremental or decremental reserve qualities. On average, about 3300 MW of tertiary reserve were traded each hour in Germany in 2003 (in comparison to the trading volume on the spot market this illustrates the importance of the reserve markets in Germany).

2.3 Trading sequence

Trading on the spot market and the (tertiary) reserve markets in Germany is sequential, i. e. an available capacity can be offered again on subsequent markets upon rejection on an earlier market (as long as defined technical requirements are fulfilled). Today, the spot market is traded between the reserve markets. The markets operated by Vattenfall Europe Transmission GmbH and E.ON Netz GmbH trade before and the markets operated by EnBW Transportnetze AG and RWE Net AG trade after the market clearing prices on the spot market are announced. However, this trading sequence is currently not subject to regulation and thus can change from time to time. It may be noted that on all the markets the respective products are traded simultaneously.

3 Bidding Methodology

In single-shot uniform-priced auction markets the bidding problem is defined by imperfections. Can this market seen to be perfect, any bidder would be a price-taker. Following microeconomic theory this would result in an optimal bidding price equal to the marginal costs. As soon as a bidder bids other than marginal costs he tries to exploit the imperfections in the market setting. Such a behavior is called strategic bidding. If the bidder can increase her profits by strategic bidding or by any means other than lowering her costs, she is said to have market power. In this paper the spot market is designed to be a uniform-priced auction. The reserve markets, however, are designed to be pay-as-bid auctions. With pay-as-bid pricing, the bidders incentive is to bid as close to the *a priori* unknown clearing price as possible. Hence, all bidders may bid higher than marginal costs with rewards to those bidders that can best guess the clearing price. Following this discussion it can be concluded that the bidding problem in auctions with pay-as-bid pricing arises even if a bidder without any market power is considered. But in this paper no single-shot auction rather a sequential trading setting is considered. Here it is important to note that a bidder has the opportunity to offer a given capacity in a subsequent auction. Then the bidder may offer a price bid higher than marginal costs in an earlier uniform-priced spot market, even though the bidder may be assumed to be a price-taker. In the following a few assumptions are considered:

A1) The bidder j is assumed to be a price-taker on the spot market but may have market power on the reserve markets.

A2) The bidder knows the method for paying the bids. A bid is defined by a bidding capacity L_j^{B} and a bidding price p_j^{B}.

A3) The capacity $L^{\max}$ to be procured by the TSO on a reserve market is defined *ex ante*. The capacity is constant, price inelastic and known to the bidders.

A4) The products, i. e. spot energy and power systems reserve, are homogeneous. This is true if the generation units fulfill defined technical requirements. The procurement decision is then based on the bidding price p_j^{B} only.

A5) The considered strategically behaving bidder is risk neutral. This assumption is justified given that the auctions take place repeatedly. The earnings at stake in each single auction are hence relatively small compared to the value-at-risk or other risk limits relevant for the bidder. The probability of repeatedly unsuccessful bidding is thereby limited given the (at least partial) independence of auction results.

A6) The publication of the market prices is transparent and characterized by the bidding capacities L_i^{B} and prices p_i^{B} of all offers $i \in \mathbb{I}$ on the reserve markets and of the market clearing price p^{S} on the spot market.

The methodology is based an the assumption that the behavior of the competing bidders can be summarized in a probability distribution of the market price. Following this general approach a probability of acceptance can be calculated leading to an expected profit to be maximized. With the bidding price p^{B} this probability of acceptance $\mathrm{P}^{\mathrm{A}}(\chi > p^{\mathrm{B}})$ may generally be calculated by (the index j for the considered bidder is omitted in the following):

$$\mathrm{P}^{\mathrm{A}}(\chi > p^{\mathrm{B}}) = 1 - F^{\chi}(p^{\mathrm{B}}) = 1 - \int_{-\infty}^{p^{\mathrm{B}}} f^{\chi}(p)\, dp \tag{1}$$

Thereby the market price is assumed to be a stochastic variable χ following the density function $f^{\chi}(p) : \mathbb{R} \mapsto \mathbb{R}_{+}$ with the probability distribution $F^{\chi}(p) : \mathbb{R} \mapsto [0,1]$. Considering the bidding capacity L^{B} and the bidding costs c^{B} the expected profit $\tilde{\Pi}$ may then be calculated by:

$$\max_{\{p^{\mathrm{B}}\}} \quad \tilde{\Pi} = \mathrm{P}^{\mathrm{A}}(\chi > p^{\mathrm{B}})\, L^{\mathrm{B}} \left(p^{\mathrm{B}} - c^{\mathrm{B}}\right) \tag{2}$$

In Subsections 3.1 and 3.2 bidding methodologies based on this general approach are presented for spot and reserve markets respectively. Thereby the methodologies are formulated for bidding a capacity L^{B} provided by a single power plant in a single product on the respective market. The approach considered for bidding on the reserve markets is described in greater detail in Swider and Weber (2005). In Subsection 3.3 these approaches are extended to consider a power plant portfolio, several products and sequential bidding.

3.1 Spot Market

The spot market is characterized by a double-sided call auction with a uniform market-clearing price. This price can generally be seen as a stochastic variable allowing to derive a continuous probability distribution. For the uniform spot market price a density function of the form $f^{\mathrm{S}}(p) : \mathbb{R}_+ \mapsto \mathbb{R}_+$ may be taken. One possibility could be a log-normal distribution:

$$f^{\mathrm{S}}(p) = \frac{1}{\sqrt{2\pi}\xi p} \exp\left(- \left(\frac{\ln(p) - \varrho}{\sqrt{2}\xi} \right)^2 \right) \tag{3}$$

The decision for the distribution function within this paper is based on an analysis of historic price data. Using time series of the spot market prices the distribution parameters ϱ and ξ in Eq. (3) can be derived using econometric methods. These parameters are directly related to the expected value μ and the standard deviation σ:

$$\xi = \sqrt{\ln((\sigma/\mu)^2 + 1)} \quad \text{and} \quad \varrho = \ln(\mu) - 0.5\xi^2 \tag{4}$$

Following Eq. (1) the probability of acceptance, considering that a bid is accepted only and entirely if the spot market price is higher than the bidding price, is calculated using the primitive of the distribution of the spot market price given in Eq. (3):

$$\mathrm{P}^{\mathrm{A}}(p^{\mathrm{S}} > p^{\mathrm{B}}) = 1 - \int_{-\infty}^{p^{\mathrm{B}}} f^{\mathrm{S}}(p)\, dp \tag{5}$$

On the considered spot market the next-days uniform market price is *ex ante* unknown. In order to estimate the expected profit it is therefore necessary to find a description of the expected spot market price. This is possible by calculating the conditional expected spot market price:

$$\mathrm{E}[p^{\mathrm{S}} | p^{\mathrm{S}} > p^{\mathrm{B}}] = \frac{1}{\mathrm{P}^{\mathrm{A}}(p^{\mathrm{S}} > p^{\mathrm{B}})} \int_{p^{\mathrm{B}}}^{\infty} p\, f^{\mathrm{S}}(p)\, dp \tag{6}$$

Considering the probability of acceptance in Eq. (5) and the conditional expected market clearing price in Eq. (6) the expected profit on a spot market $\tilde{\Pi}^{\mathrm{S}}$ can be calculated by (with the marginal costs $c^{\mathrm{S}} \leq p^{\mathrm{B}}$):

$$\tilde{\Pi}^{\mathrm{S}} = \mathrm{P}^{\mathrm{A}}(p^{\mathrm{S}} > p^{\mathrm{B}})\, L^{\mathrm{B}} \left(\mathrm{E}[p^{\mathrm{S}} | p^{\mathrm{S}} > p^{\mathrm{B}}] - c^{\mathrm{S}} \right) \tag{7}$$

Even though energy is traded on the spot market the bidding capacity is considered in Eq. (7). This is possible as long as hourly products are assumed to be traded.

3.2 Reserve Market

In a pay-as-bid procurement auction no uniform market price rather a price range can be observed. In fact the market price can be between the efficiency and the marginal price. The efficiency price p^{E} is thereby set by the less and the marginal price p^{M} by the most expensive accepted bid (as seen from the procurer's perspective). If the accepted offers are transparently published these characteristic market prices can be derived *ex post* by analysing the merit order of accepted offers. However, at the time of submitting a bid the efficiency price, the marginal price and the cascaded merit order curve are *ex ante* unknown. Hence, a solution on how to deal with these uncertainties needs to be found.

Following Assumption A6 historic time series of bidding capacities and prices are assumed to be available to the bidder. Hence, probability distributions of the efficiency and marginal prices, with the prices assumed to be stochastic variables, can easily be derived. For the efficiency price a density function of the form $f^{\mathrm{E}}(p) : \mathbb{R} \mapsto \mathbb{R}_+$ may be taken. One possibility could be a Gaussian-mixture distribution:

$$f^{\mathrm{E}}(p) = \sum_{j=1}^{m} \frac{\lambda_j}{\sqrt{2\pi}\sigma_j} \exp\left(- \left(\frac{p - \mu_j}{\sqrt{2}\sigma_j} \right)^2 \right) \tag{8}$$

Eq. (8) gives a mixture of m normal distributions. The distribution is characterized by the expected values μ_j, the standard deviations σ_j and the probabilities λ_j (for the latter holds $\sum_{j=1}^{m} \lambda_j = 1$). Within this paper a mixture of two normal distributions ($m = 2$) is taken.

To estimate the density function of the marginal price, first a density function of the difference between the marginal and efficiency price needs to be calculated. For this difference a density function of the form $f^{\Delta\mathrm{ME}}(p) : \mathbb{R}_+ \mapsto \mathbb{R}_+$ may be taken. One possibility could be an Erlang distribution:

$$f^{\Delta\mathrm{ME}}(p) = \frac{p}{b^2} \exp\left(-\frac{p}{b} \right) \tag{9}$$

Knowing these basic distributions the probability density function $f^{\mathrm{M}}(p) : \mathbb{R} \mapsto \mathbb{R}_+$ of the marginal price can be calculated by applying a single-sided convolution:

$$f^{\mathrm{M}}(p) = \int_{0}^{\infty} f^{\mathrm{E}}(p-u)\, f^{\Delta\mathrm{ME}}(u)\; du \tag{10}$$

Considering available time series of the efficiency and marginal prices the price distribution parameters in Eq. (8) and (9) can be derived using econometric methods. The decision for the distribution functions within this paper is based on an analysis of available historic price data and, in difference to Swider and Weber (2005), the important fact that with these functions the convolution in Eq. (10) can be found analytically (cf. Swider, 2005b).

So far was shown that characteristic market prices, i. e. efficiency and marginal prices, can be seen as stochastic variables and be represented by probability distributions. However, to follow the fundamental approach presented above the probability of acceptance need to be calculated. It may be first noted that one's bidder's offer will only be accepted if the bidding price is lower than a relevant market price. Thereby the relevant market price $p^{\mathrm{R}} \in [p^{\mathrm{E}}, p^{\mathrm{M}}]$ describes the price on the merit order of all offers necessary to displace competing bidders.

Hence, given a bidding price p^{B} the capacity procured depends on the bidding capacity L^{B} and the *ex ante* unknown and unpredictable cascaded merit order. For the merit order a linear approximation between the efficiency and marginal price is assumed. For the relevant market price p^{R} then follows:

$$p^{\mathrm{R}} = \left(p^{\mathrm{M}} - p^{\mathrm{E}}\right) k(L^{\mathrm{B}}) + p^{\mathrm{E}} \tag{11}$$

With $k(L^{\mathrm{B}}) \in [0,1]$ as the index of the merit order:

$$k(L^{\mathrm{B}}) = \frac{L^{\max} - L^{\mathrm{B}}}{L^{\max} - L^{\min}} \tag{12}$$

Following the discussion above, the relevant market price, cf. Eq. (11), needs to be seen as a stochastic variable. Hence, Eq. (10) needs to be extended resulting in a density function of the relevant market price p^{R} depending on the bidding capacity L^{B}:

$$f^{\mathrm{R}}(p^{\mathrm{B}}\,; L^{\mathrm{B}}) = \int_{0}^{\infty} f^{\mathrm{E}}(p^{\mathrm{B}} - k(L^{\mathrm{B}})\, u)\, f^{\Delta\mathrm{ME}}(u)\; du \tag{13}$$

Following Eq. (1) the probability of acceptance, considering that a bid is accepted only and entirely if the relevant market price is higher than the bidding price, is calculated using the primitive of the distribution of the relevant market price given in Eq. (13):

$$\mathrm{P}^{\mathrm{A}}(p^{\mathrm{R}} > p^{\mathrm{B}}\,; L^{\mathrm{B}}) = 1 - \int_{-\infty}^{p^{\mathrm{B}}} f^{\mathrm{R}}(p\,; L^{\mathrm{B}})\; dp \tag{14}$$

The analysis so far has dealt with the market prices from a systems perspective with the efficiency and marginal prices described as stochastic processes. Following Assumption A1 a non-competitive reserve market is considered. Hence a single bidder with a non-negligible bid quantity has to take into account that her bid may influence the market price – not only in the bidding period itself but also in subsequent periods. Thereby, most problematic from a bidder's point of view are decreases in the future average price level as a consequence of his own bid. Such a long-term price effect of price dumping is most likely to occur, if the price bid becomes the efficiency price.

If the process of the efficiency price can be described by an ARMA model it can be shown that the long-term unity price effect due to price dumping ν can be calculated by analysing the step response on this "efficiency price system" (cf. Swider, 2005b). Considering the conditional expected efficiency price

$$\mathrm{E}[p^{\mathrm{E}}|p^{\mathrm{E}} > p^{\mathrm{B}}] = \frac{1}{\mathrm{P}[p^{\mathrm{E}} > p^{\mathrm{B}}]} \int_{p^{\mathrm{B}}}^{\infty} p\; f^{\mathrm{E}}(p)\; dp \tag{15}$$

the probability weighted average of the decrease of the efficiency price through the bidding price $\Delta\tilde{p}^{\nu}(p^{\mathrm{B}})$ can be determined by multiplying the price dumping effect ν with the expected height of the initial shock by:

$$\Delta\tilde{p}^{\nu}(p^{\mathrm{B}}) = \nu\, \mathrm{P}[p^{\mathrm{E}} > p^{\mathrm{B}}] \left(p^{\mathrm{B}} - \mathrm{E}[p^{\mathrm{E}}|p^{\mathrm{E}} > p^{\mathrm{B}}]\right) \tag{16}$$

With the probability of acceptance, cf. Eq. (14), and the long-term price effect of price dumping, cf. Eq. (16), the expected profit $\tilde{\Pi}^{\mathrm{R}}$ is given by:

$$\tilde{\Pi}^{\mathrm{R}} = \mathrm{P}^{\mathrm{A}}(p^{\mathrm{R}} > p^{\mathrm{B}}\,; L^{\mathrm{B}})\, L^{\mathrm{B}} \left(p^{\mathrm{B}} + \Delta\tilde{p}^{\nu}(p^{\mathrm{B}}) - c^{\mathrm{B}}\right) \tag{17}$$

In Eq. (17) also the expected bid costs c^{B}, i. e. the costs occurring when the bid is accepted, are relevant.

The methodologies presented so far focused on bidding on one market, a spot and a reserve market respectively. However, in many electricity markets a bidder has the opportunity to decide on the capacity and price to offer on several spot and reserve markets. Within this paper trading on these markets is assumed to be sequential, i. e. a capacity offered on an earlier market may be offered on a subsequent market upon rejection on the earlier one.

The general approach for sequential bidding already discussed in Section 1 leads to calculate the expected bidding capacity by considering the probability of rejection of an offer in an earlier auction. Following that discussion: If a bidder bids the capacity L_1^{B} with price p_1^{B} in the first auction, she expects the bid to be accepted with the probability $\mathrm{P}^{\mathrm{A}}(p_1^{\mathrm{M}} > p_1^{\mathrm{B}})$. In the second auction she may wish to bid L_2^{B} plus the expected rejected capacity on the first market given by $\tilde{L}_1^{\mathrm{R}} = \mathrm{P}^{\mathrm{R}}(p_1^{\mathrm{M}} \leq p_1^{\mathrm{B}})\, L_1^{\mathrm{B}}$ with price p_2^{B}. Hence the expected bidding capacity $\tilde{L}_2^{\mathrm{B}}$ on the second market equals to the sum $\tilde{L}_2^{\mathrm{B}} = L_2^{\mathrm{B}} + \tilde{L}_1^{\mathrm{R}}$. Thus it follows $\tilde{L}_2^{\mathrm{B}} \geq L_2^{\mathrm{B}}$. Note that if the equality holds the results will correspond to a simultaneous trading setting.

The expected bidding capacity on market m in a sequence of several markets $m \in \mathbb{M}$ is generally given by:

$$\tilde{L}_m^{\mathrm{B}} = \sum_{n=0}^{m-1} \left[L_{m-n}^{\mathrm{B}} \prod_{i=1}^{n} \mathrm{P}_{m-i}^{\mathrm{R}}(p_{m-i}^{\mathrm{M}} \leq p_{m-i}^{\mathrm{B}}) \right] \tag{18}$$

One important question to be answered is: Can this general approach be applied if sequential bidding on spot and reserve markets is considered? The general approach implicitly assumes that the accepted capacity on a market can be seen as a continuous function of the actual market clearing price. This may be illustrated considering a simple example. If the bidder plans to bid $L^{\mathrm{B}} = 200$ MW equally on two sequentially traded markets and the probability of acceptance on the first market is given by $\mathrm{P}^{\mathrm{A}}(p_1^{\mathrm{M}} > p_1^{\mathrm{B}}) = 0.5$ the question is: What bidding capacity need to be considered on the second market?

Following the approach discussed above this capacity simply equals to the expected bidding capacity resulting to $\tilde{L}_2^{\mathrm{B}} = L^{\mathrm{B}}/2 + \tilde{L}_1^{\mathrm{R}} = 150$ MW. But actually the bidding capacity will either equal $L_2^{\mathrm{B}} = 100$ MW or $L_2^{\mathrm{B}} = 200$ MW respectively upon acceptance or rejection of the offer on the first market. This leads to a stochastic tree representing this wait-and-see structure, cf. Figure 1 (a).

As long as the function of profit over bidding capacity can be seen to be a concave function the assumption of considering the expected bidding capacity may result in an overestimated expected profit and hence an overvaluation of the rejected capacity on the earlier market in a sequence of several. This overvaluation greatly depends on the curvature of the function of the expected profit, cf. Figure 1 (b). However, by applying the methods for bidding on spot and reserve markets as presented in Section 3.1 and 3.2 can be shown that the functions of profit over bidding capacity may sufficiently be approximated by a linear trend in the relevant areas of the bidding capacity (cf. Swider, 2005b). Especially as long as the bidding capacity is far less than the actual demand, i. e. $L^{\mathrm{B}}_m \ll L^{\max}_m$. Hence, the general approach for sequential bidding as discussed above is taken within this paper.

To consider sequential bidding on spot and reserve markets the proposed methods and especially Eq. (7) and (17) need to be extended. Following Assumption A1 a competitive spot market leads to consider just one such market m^{S} (more than one would result in bidding on the market with the highest expected profit). The assumption of non-competitive reserve markets on the other hand leads to consider several such markets as strategic bidding is likely to occur, i. e. $\mathrm{m}^{\mathrm{R}}_m \in \mathbb{M}^{\mathrm{R}} = \{\mathrm{m}^{\mathrm{R}}_1, \mathrm{m}^{\mathrm{R}}_2, \ldots, \mathrm{m}^{\mathrm{R}}_M\}$. The set of all markets is given by $\mathbb{M} = \mathbb{M}^{\mathrm{R}} \cup \mathrm{m}^{\mathrm{S}}$ (in the following the sequence of the markets needs to be taken into account).

For the spot and reserve markets different products are distinguished, one-hour products $o^{\mathrm{S}} \in \mathbb{O}^{\mathrm{S}}$ and several-hour products $o^{\mathrm{R}}_m \in \mathbb{O}^{\mathrm{R}}_m$ respectively. The set of all reserve markets is given by $\mathbb{O}^{\mathrm{R}}$ and of all products by $\mathbb{O} = \mathbb{O}^{\mathrm{S}} \cup \mathbb{O}^{\mathrm{R}}$. The duration of the products is defined by the binary function $H_o(h) : [1, 24] \mapsto \{0, 1\}$. This binary function takes the value 1 if the product $o \in \mathbb{O}$ is defined in the respective hour $h \in [1, 24]$ and the value 0 otherwise. The product times on any market are not allowed to overlap and in sum need to cover a duration of 24 hours (note that e. g. the product times of reserve market 1 may unequally overlap with the product times of reserve market 2). For each product separate and uncorrelated stochastic price processes are assumed. This allows to derive the probability density functions as given by Eq. (3), (8), (9) and (13).

The available bidding capacities $L^{\mathrm{B}}_{o,k}$ are defined by a portfolio of power plants $k \in \mathbb{K}$. The maximal producing capacity indexed by power plant k and hour h is represented with $L^{\mathrm{P}_2}_{k,h}$. To maintain a certain degree of simplicity no complete unit-commitment is considered within this paper. Rather a preplanning is assumed resulting in a producing capacity $L^{\mathrm{P}_0}_{k,h} \geq 0$ of the available power plants in each hour. The product specific bidding capacities on the reserve markets are defined to be the sum over the power plants, i. e. $L^{\mathrm{B}}_o = \sum_{k \in \mathbb{K}} L^{\mathrm{B}}_{o,k}$. The boundaries of the bidding capacities are not only given by the available capacities but are also subject to products and given by minimal and maximal capacities. On the reserve market the lower boundary is set to be zero and the upper to a maximal capacity $L^{\max}_o$ to be procured. On the spot market no upper boundary is set, whereas the lower boundary is also set to be zero.

In the following Eq. (7) need to be extended to consider the products $o \in \mathbb{O}^{\mathrm{S}}$ and power plants $k \in \mathbb{K}$ to calculate one's bidder's expected profit on the spot market. Therefore, the following variables have to be replaced: $p^{\mathrm{S}} = p^{\mathrm{S}}_{o}$, $p^{\mathrm{B}} = p^{\mathrm{B}}_{o,k}$ and $c^{\mathrm{S}} = c^{\mathrm{S}}_{o,k}$. Additionally the bidding capacity, in Eq. (7) originally indicated by L^{B}, need to be replaced. Here the expected bidding capacity $\tilde{L}^{\mathrm{B}}_{o,k} \geq L^{\mathrm{B}}_{o,k}$ is needed as sequential bidding is assumed (the calculation will be discussed below). This results to:

$$\tilde{\Pi}^{\mathrm{S}}_{m} = \sum_{k\,\in\,\mathbb{K}} \sum_{o\,\in\,\mathbb{O}^{\mathrm{S}}} \mathrm{P}^{\mathrm{A}}(p^{\mathrm{S}}_{o} > p^{\mathrm{B}}_{o,k})\, \tilde{L}^{\mathrm{B}}_{o,k} \left(\mathrm{E}[p^{\mathrm{S}}_{o}|p^{\mathrm{S}}_{o} > p^{\mathrm{B}}_{o,k}] - c^{\mathrm{S}}_{o,k}\right) \tag{19}$$

The marginal generation costs can be calculated by:

$$c^{\mathrm{S}}_{k,h}(L^{\mathrm{P}_1}_{k,h},\, L^{\mathrm{P}_0}_{k,h}) = \frac{Q(L^{\mathrm{P}_1}_{k,h}) - Q(L^{\mathrm{P}_0}_{k,h})}{L^{\mathrm{P}_1}_{k,h} - L^{\mathrm{P}_0}_{k,h}} \cdot c^{\mathrm{F}}_{k} \tag{20}$$

Thereby, the producing capacity $L^{\mathrm{P}_0}_{k,h}$ after preplanning and, considering accepted offers on the spot market, the capacity $L^{\mathrm{P}_1}_{k,h} = L^{\mathrm{P}_0}_{k,h} + L^{\mathrm{B}}_{o,k}\, H_{o}(h)$ is needed. The non-linear fuel consumption curve, multiplied by the fuel costs c^{F}_{k}, can be given by ($\varsigma_i = \mathrm{const}$):

$$Q(L^{\mathrm{P}}_{k,h}) = \varsigma_0 + \varsigma_1 L^{\mathrm{P}}_{k,h} + |\varsigma_2| \left(L^{\mathrm{P}}_{k,h}\right)^2 \tag{21}$$

In the following Eq. (17) need to be extended to consider the products $o \in \mathbb{O}^{\mathrm{R}}_{m}$ and power plants $k \in \mathbb{K}$ to calculate one's bidder's expected profit on a reserve market. Therefore, the following variables have to be replaced: $p^{\mathrm{R}} = p^{\mathrm{R}}_{o}$ and $p^{\mathrm{B}} = p^{\mathrm{B}}_{o}$ ($p^{\mathrm{B}}_{o,k} = p^{\mathrm{B}}_{o} = \mathrm{const}\ \forall\ k \in \mathbb{K}$). In line with the discussion above again also the bidding capacity need to be replaced. Contrarily to the costs considered for bidding on the spot market any bid costs on the reserve markets are neglected as the procured reserve is seldom actually used (cf. Swider and Weber, 2003; Swider, 2005b). This results to:

$$\tilde{\Pi}^{\mathrm{R}}_{m} = \sum_{k\,\in\,\mathbb{K}} \sum_{o\,\in\,\mathbb{O}^{\mathrm{R}}_{m}} \mathrm{P}^{\mathrm{A}}(p^{\mathrm{R}}_{o} > p^{\mathrm{B}}_{o}; \textstyle\sum_{k\,\in\,\mathbb{K}} \tilde{L}^{\mathrm{B}}_{o,k})\, \tilde{L}^{\mathrm{B}}_{o,k} \left(p^{\mathrm{B}}_{o} + \Delta\tilde{p}^{\nu}_{o}(p^{\mathrm{B}}_{o})\right) \tag{22}$$

As already mentioned, the products on the considered reserve markets are defined to cover different durations of time that may hence unequally overlap. This aspect need to be taken into account in order to calculate the expected bidding capacity $\tilde{L}^{\mathrm{B}}_{o,k}$ and is not reflected by Eq. (18). The calculation is however possible by estimating the minimal expected bidding capacity available during the respective product time (note that by calculating the expected bidding capacity the respective trading sequence is of major importance). The expected bidding capacity on market m in a sequence of several markets

$m \in \mathbb{M}$ is given by:

$$\tilde{L}^{\mathrm{B}}_{o_m,k} = \min_{h \,|\, H_{o_m}(h)=1} \left\{ \sum_{n=0}^{m-1} \left[L^{\mathrm{B}}_{o_{m-n},k \,|\, H_{o_{m-n}}(h)=1} \prod_{i=1}^{n} \mathrm{P}^{\mathrm{R}}_{o_{m-i},k \,|\, H_{o_{m-i}}(h)=1} \right] \right\} \tag{23}$$

Thereby the probability of rejection depends on the respective market:

$$\mathrm{P}^{\mathrm{R}}_{o,k} = \begin{cases} \mathrm{P}^{\mathrm{R}}(p^{\mathrm{S}}_{o} \leq c^{\mathrm{S}}_{o,k}) & \text{, if } o \in \mathbb{O}^{\mathrm{S}} \\ \mathrm{P}^{\mathrm{R}}(p^{\mathrm{R}}_{o} \leq p^{\mathrm{B}}_{o}; \sum_{k \in \mathbb{K}} \tilde{L}^{\mathrm{B}}_{o,k}) & \text{, if } o \in \mathbb{O}^{\mathrm{R}}_{m} \end{cases} \tag{24}$$

This discussion finally results in a method for sequential bidding, where the bidding capacities and prices on the reserve markets and on the spot market are calculated by maximizing a stochastic non-linear objective function of expected profit given by:

$$\begin{aligned} &\max_{\left\{p^{\mathrm{B}}_{o,k},\, L^{\mathrm{B}}_{o,k}\right\}} \quad \tilde{\Pi} = \sum_{k \in \mathbb{K}} \sum_{m \in \mathbb{M}} \left(\tilde{\Pi}^{\mathrm{S}}_{m} + \tilde{\Pi}^{\mathrm{R}}_{m} \right) \\ &\text{s. t.} \sum_{k \in \mathbb{K}} \tilde{L}^{\mathrm{B}}_{o,k} \leq L^{\max}_{o} \;\; ; \;\; L^{\mathrm{B}}_{o,k} \geq 0 \\ &\qquad \sum_{o \in \mathbb{O}} L^{\mathrm{B}}_{o,k} \, H_{o}(h) \leq L^{\mathrm{P}_2}_{k,h} \;\; ; \;\; p^{\mathrm{B}}_{o,k} \geq 0 \end{aligned} \tag{25}$$

4 Exemplary Application

It has been mentioned that no complete unit-commitment is considered within this paper in order to maintain a certain degree of simplicity. The exemplary application is therefore simplified by assuming a preplanning of a defined exemplary power plant portfolio. With this simplification it is possible to neglect e. g. availabilities, start-up costs and minimum operation times.

The results of the non-linear objective function in Eq. (25) are derived with Matlab® and the `fmincon` function, based on a sequential quadratic programming approach, of the optimization toolbox.

To be able to show the applicability of the methods in Subsection 4.1 the markets and products to be considered and in Subsection 4.2 a power plant portfolio is described. This is followed by a discussion of the main results in Subsection 4.3.

4.1 Markets and Products

The application is based on markets operating in Germany. Thereby one spot market, operated by the European Energy Exchange AG (EEX), and two reserve markets, respectively operated by the RWE Net AG and the E.ON Netz GmbH, are considered. In the following the historic trading sequence E.ON-EEX-RWE is considered. The results are compared to a simultaneous trading setting, i. e. a capacity can be offered on one market only. With the methodology presented above this is possible by considering the bidding capacity $L^{\mathrm{B}}_{o,k}$ instead of the expected bidding capacity $\tilde{L}^{\mathrm{B}}_{o,k}$ in Eq. (19), (22) and (25).

On these markets different product times are distinguished, one-hour products on the spot and several-hour products on the reserve markets. On the RWE reserve market five products represented by $\mathbb{O}^{\mathrm{R}}_{3}$ are traded. Except product o_3 all products cover time periods of four hours: product o_1 from 0-4 am, product o_2 from 4-8 am, product o_3 from 8 am-4 pm, product o_4 from 4-8 pm and product o_5 from 8-12 pm. On the EEX spot market 24 hourly products represented by $\mathbb{O}^{\mathrm{S}}_{2}$ are traded. On the E.ON reserve market two products represented by $\mathbb{O}^{\mathrm{R}}_{1}$ are traded: one peak product o_6 covering the time period from 6 am-10 pm and one base product o_7 covering all other hours. The reserve markets are characterized by the maximal procuring capacities. Here they are set to be $L^{\max}_{o} = 750$ MW $\forall\, o \in \mathbb{O}^{\mathrm{R}}_{3}$ and $L^{\max}_{o} = 1100$ MW $\forall\, o \in \mathbb{O}^{\mathrm{R}}_{1}$.

For the products the parameters of the respective probability density functions as needed in Eq. (3), (8), (9) and (13) are given in Table 1 and 2, respectively for the spot and reserve market. The parameters have been estimated using publicly available time series of the relevant market prices. The estimation of the parameters and the day-ahead forecasts of the expected prices are based on econometric methods applying the ARMA approach (cf. Swider, 2005b).

4.2 Power Plant Portfolio

The exemplary power plant portfolio is defined according to reflect a small generation company. In the portfolio two coal fired (k_1 and k_2), one pumped hydro (k_3) and two gas fired power plants (k_4 and k_5) are considered. The characteristics of the plants are given in Table 3. Thereby the maximal producing capacity, the parameters of the fuel consumption curve, cf. Eq. (21), and the fuel costs, needed for calculating the marginal costs of generation cf. Eq. (20), are given. The latter have been derived using statistics of the German electricity system (cf. Swider, 2005b, and the references therein).

It is assumed that the bidder needs to cover a contracted and inflexible demand. The demand is partly covered by the exemplary power plant portfolio following a least-cost preplanning unit-commitment not described in this paper. Following this assumption some of the power plants are expected to operate in part-load in different moments of time with the remaining capacities available to bid on the considered markets. The resulting available capacities for sequential bidding are given in Figure 2.

4.3 Results

In an analysis of historic price data of the considered spot and reserve markets may be seen that the expected profits on the reserve markets are generally higher than on the spot market (cf. Swider and Weber, 2003). Thereby the expected profits on the E.ON market are higher than on the RWE market, cf. Table 2. This may lead to the initial assumption that nearly all of the available bidding capacities will be offered on the E.ON reserve market. However, this assumption neglects the reserve markets to be non-competitive.

Hence, optimized bidding on the reserve markets will not always result in offering the available capacity entirely on the market with the highest expected profit. This is due to the presented methodology with the probability of acceptance depending on the bidding capacity, cf. Eq. (14). With an in-depth analysis of the methodology may be seen that the function of expected profit over the bidding capacity is concave and monotonic increasing (cf. Swider, 2005b). From this follows *a priori* that the optimal bidding capacity equals the maximal one. However, considering the function being concave a non-proportional increase of the expected profit with increasing bidding capacity can be observed. This implies that it might be advisable to bid only a part of the total available bidding capacity in one market.

Nevertheless, by applying the methodology for simultaneous bidding and considering the described products and the defined power plant portfolio it can be shown that high capacities are offered on the E.ON reserve market, cf. Table 4 and Figure 3. However, it can be seen that in several hours bidding on the spot market is superior to bidding on the reserve markets. This is partly due to the already discussed concave function of expected profit. Another reason is a relatively low difference between the expected spot market prices and the marginal generation costs in these hours. It can, hence, be shown that the methodology reflects that power plants with high marginal generation costs (k_4 and k_5) are predominantly offered on the reserve markets.

The high bidding capacity on the E.ON reserve market can also be seen by applying the methodology for sequential bidding and considering the described products and the defined power plant portfolio, cf. Table 4 and Figure 4.

Thereby the E.ON reserve market is the first market in the trading sequence, followed by the EEX spot and the RWE reserve market. It is most important to see that the methodology reflects the *a priori* expectation of higher bidding capacities in the reverse rank number of the markets. Hence, the initial capacities reserved for bidding on the RWE reserve market, the third market in the trading sequence, at the time of the first trading decision (for bidding on the E.ON reserve market) are relatively low, cf. Figure 4. However, the expected bidding capacities, taking the expected rejected capacities under account, are higher. This can be shown analyzing the expected bidding capacities on the markets in Figure 5 to 7. The expected bidding capacities on the first market do evidently equal the bidding capacities as no prior market is assumed to be traded, cf. Figure 5 for the E.ON market. On the second and third market, however, the expected rejected capacities offered on the respective prior market are taken into account, cf. Figure 6 and Figure 7 for the EEX spot and the RWE reserve market respectively.

But not only higher bidding capacities also higher bidding prices in the reverse rank number of the markets may be seen, cf. Table 4 and especially Figure 8. The bidding prices tend to be higher on prior markets in the sequential compared to the simultaneous trading setting. This generally results in a lower probability of acceptance and thus increases the probability of higher bidding capacities on subsequent markets. Figure 8 (a) shows the supply curves of the marginal costs as estimated applying Eq. (20) and the optimized bidding prices on the EEX spot market, the second market in the trading sequence. Figure 8 (b) shows the respective probabilities of acceptance. It may be seen that in several hours of the trading day the optimized bidding prices on the EEX spot market are much higher than the marginal costs (that states the lower boundary). This results in lower probabilities of acceptance and hence higher expected bidding capacities on the subsequent RWE reserve market. This is mainly due to the profit on the reserve market expected to be higher.

It may be noted that next to the optimized bidding capacities and prices also the expected efficiency and marginal prices are given in Table 4. It may be seen that the optimized bidding prices on the RWE market are generally in-between the expected efficiency and marginal prices. The optimized bidding prices on the E.ON market on the other hand may be seen to be lower in the simultaneous and in-between in the sequential trading setting. This difference in bidding is due to the considered trading setting but also to the estimated value of the price dumping effect ν that has been estimated to be higher for the products on the RWE than on the E.ON market, cf. Table 2. Here may additionally be worth to note that the expected efficiency and marginal prices do not necessarily equal to the actual values on the respective trading day. The latter depend on the bidding behavior of the competitors. However, the presented methodology accounts for deviations between the expected values and the actual prices by considering the complete price distribution.

The maximized expected overall profit of 23726 € in the simultaneous and 25622 € in the sequential trading setting is dominated by the reserve market products and especially by the peak-product o_6 on the E.ON market. Therefore, the expected profit on the reserve markets is higher than on the spot market, 18778 € and 4948 € in the simultaneous and 21677 € and 3945 € in the sequential trading setting respectively. The low fraction of the expected spot market profit is due to relatively high marginal generation costs compared to the expected spot market prices (even though the power plants with the lowest marginal generation costs are offered on the spot market). In this case the respective offers are characterized by relatively low probabilities of acceptance and this hence results in minor expected profits.

5 Conclusions

In this paper a methodology is presented that enables a strategically behaving bidder to estimate the profit maximizing offers on sequentially traded day-ahead auction markets for spot energy and power systems reserve. The methodology is based on deriving appropriate probability density functions of the relevant market prices. Thereby special focus is given on bidding on non-competitive reserve markets designed as pay-as-bid procurement auctions. The applicability is discussed using exemplary data. It is shown that the methodology accounts for the interdependencies between diverse products on several markets and that the trading sequence leads to increasing bidding capacities and prices in the reverse rank number of the markets. Hence, the consideration of a defined trading sequence greatly influences the mathematical representation of the optimal bidding behavior under price uncertainty. Further research may lead to implement the developed methods in unit-commitment approaches and to derive optimized sequential bidding results applying a game theoretic approach.

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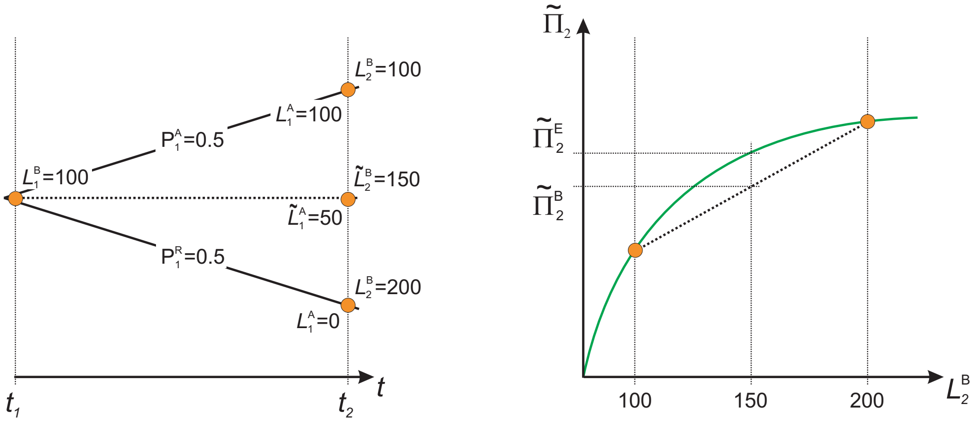
Figures and Tables



(a) Decision tree (b) Expected profit

Fig. 1. Illustration of stochastic programming in case of sequential bidding

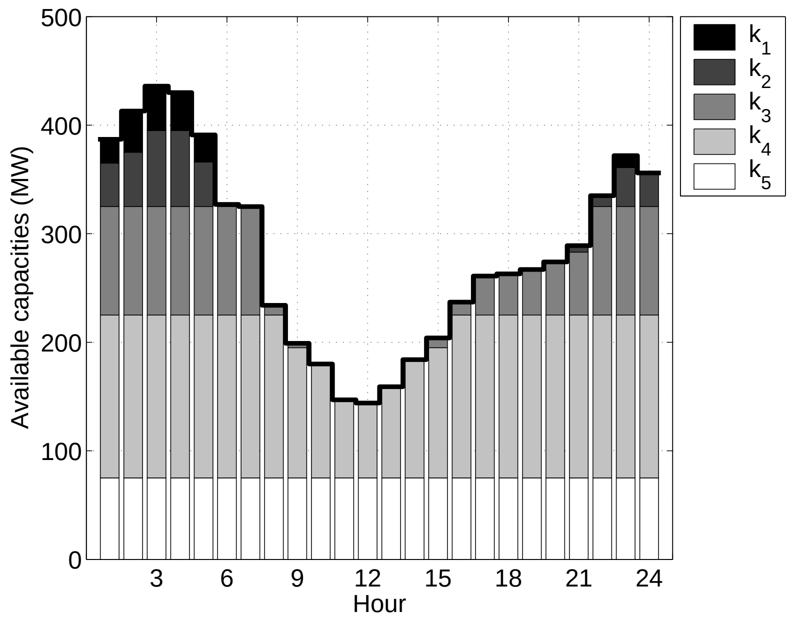


Fig. 2. Available capacities after preplanning

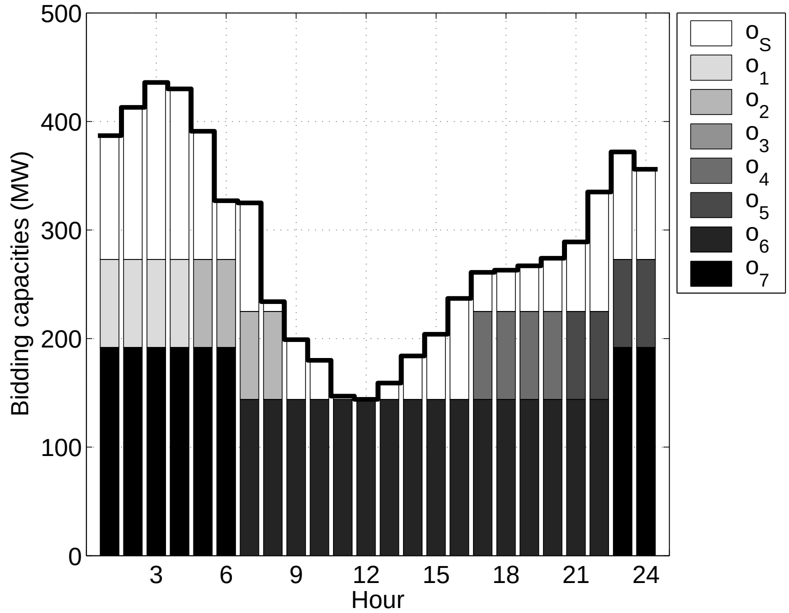


Fig. 3. Bidding capacities in the simultaneous trading setting

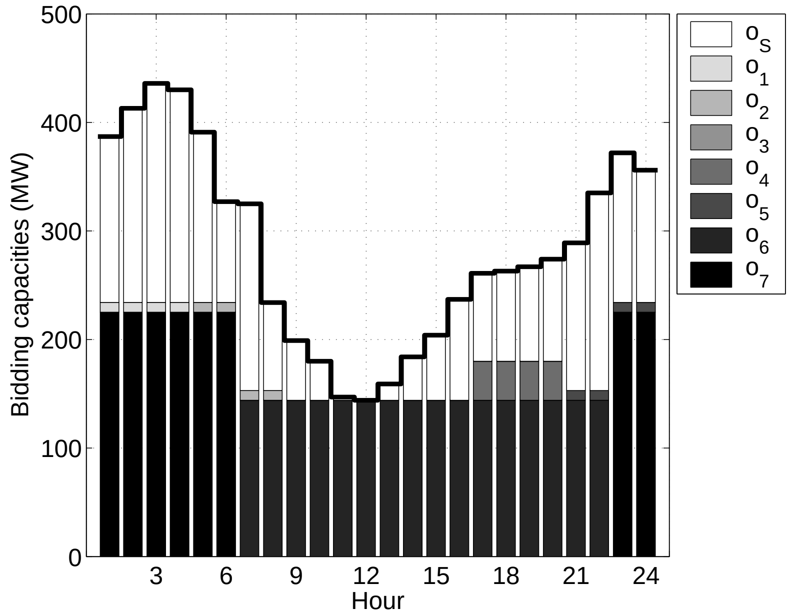


Fig. 4. Bidding capacities in the sequential trading setting

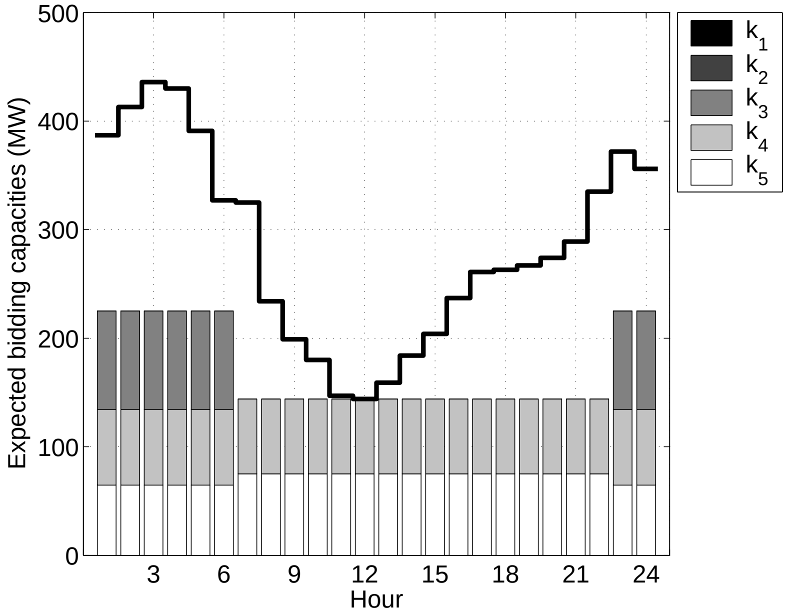


Fig. 5. Expected bidding capacities on the first market in the sequential trading setting (the E.ON reserve market)

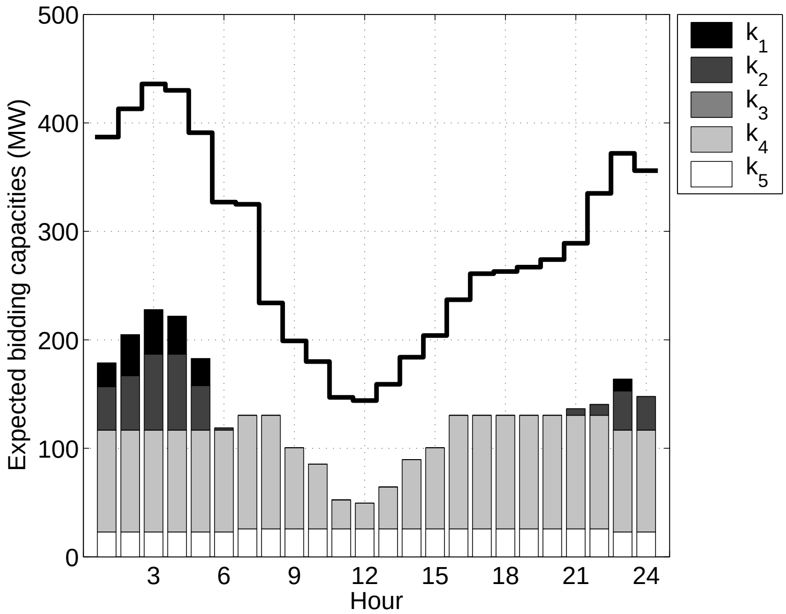


Fig. 6. Expected bidding capacities on the second market in the sequential trading setting (the EEX spot market)

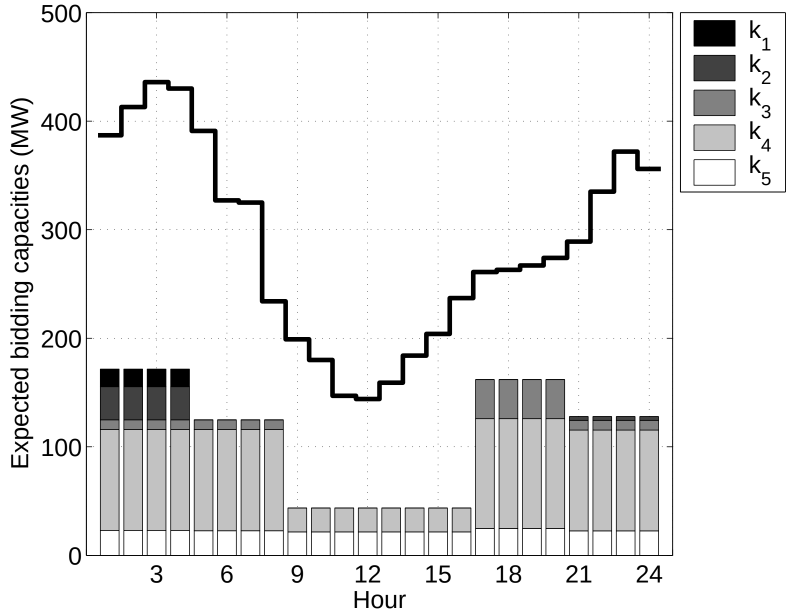
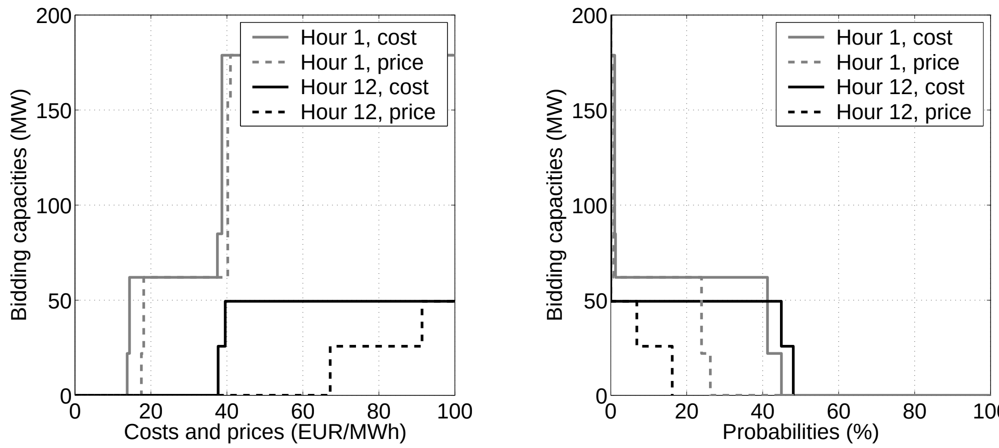


Fig. 7. Expected bidding capacities on the third market in the sequential trading setting (the RWE reserve market)



(a) Marginal costs and bidding prices

(b) Probabilities of acceptance

Fig. 8. Supply curves on the EEX spot market in the sequential trading setting

Table 1
Parameters of the EEX spot market probability density functions for the products $o \in \mathbb{O}^{\mathrm{S}}$, μ and σ in (€/MWh)

	μ	σ		μ	σ
o_1	14.5	7.3	o_{13}	30.8	19.4
o_2	10.0	7.5	o_{14}	28.8	16.9
o_3	9.1	7.7	o_{15}	28.2	15.3
o_4	8.2	6.5	o_{16}	26.0	12.4
o_5	9.9	7.4	o_{17}	22.8	9.4
o_6	15.6	7.7	o_{18}	22.5	9.1
o_7	18.2	8.2	o_{19}	22.1	9.3
o_8	27.4	14.2	o_{20}	21.2	9.3
o_9	28.2	14.2	o_{21}	21.8	9.8
o_{10}	28.8	16.6	o_{22}	20.5	10.4
o_{11}	32.3	18.9	o_{23}	18.5	8.4
o_{12}	44.2	30.1	o_{24}	13.7	5.6

Table 2
Parameters of the RWE and E.ON reserve market probability density functions for the products $o \in \mathbb{O}^{\mathrm{R}}$, μ_j and σ_j in (€/MW)

	o_1	o_2	o_3	o_4	o_5	o_6	o_7
λ_1	0.2	0.3	0.2	0.2	0.3	0.3	0.4
λ_2	0.8	0.7	0.8	0.8	0.7	0.7	0.6
μ_1	6.5	13.7	108.2	27.7	9.4	126.2	15.5
μ_2	7.1	14.1	108.8	28.3	9.8	126.6	15.6
σ_1	3.6	2.7	27.3	4.9	2.3	17.4	1.7
σ_2	0.1	0.1	1.1	0.7	0.1	2.6	0.3
ν	11.3	10.8	13.4	12.6	10.9	2.6	3.4
b	0.4	0.6	0.6	0.4	0.5	0.8	0.6

Table 3
Exemplary power plant portfolio

		k_1	k_2	k_3	k_4	k_5
L^{P_2}	(MW)	150	100	100	150	75
ς_0	(MW)	28.0	31.7	5.3	19.5	60.3
ς_1	(–)	2.18	2.13	0.98	2.30	2.19
ς_2	(MW^{-1} 10^{-3})	1.56	3.76	1.63	0.58	4.00
c^{F}	(€/MWh)	5.3	5.3	15.0	16.4	16.4

Table 4
Expected efficiency, marginal and bidding prices (€/MW) as well as expected bidding capacities (MW) as estimated for the products on the RWE and E.ON reserve markets in the simultaneous (sim) and sequential (seq) trading setting

		o_1	o_2	o_3	o_4	o_5	o_6	o_7
p^{E}	–	7.0	14.0	108.7	28.2	9.7	126.5	15.6
p^{M}	–	7.8	15.2	109.9	29.0	10.7	128.1	16.8
p^{B}	sim	7.2	14.2	–	28.2	9.9	122.9	15.5
p^{B}	seq	7.2	14.2	108.2	28.2	9.9	126.4	15.7
L^{B}	sim	81	81	–	81	81	144	192
L^{B}	seq	172	125	44	162	128	144	225