Efficient Scoring-Rule in Multi-Part Procurement Auctions for Power Systems Reserve

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***Abstract*— In this paper scoring-rules are analyzed based on experiences with markets for power systems reserve in Germany. For paying the accepted offers on these markets the pay-as-bid settlement-rule is applied. Thereby each offer consist of a reserve capacity and two prices. The capacity price is for holding the offered capacity in reserve and the energy price is for delivery in case of actual use. In today's daily operation a simple scoring-rule is applied in order to value the two-part price bids. This scoring-rule is based on procuring the offers in the rank order of the capacity prices. A first analysis of historic price data indicates efficiency problems due to this scoring-rule. This leads to propose a scoring-rule that values the two-part price bids based on the ex post knowledge of the actual use of the procured reserve. It is shown that in contrast to today's scoring-rule this duration curve approach can give incentives to reveal the marginal generation costs in the energy price bids.**

***Index Terms*— market design, pay-as-bid, power systems reserve, procurement auction, scoring-rule.**

I. Introduction

THE procurement of power systems reserve is based on the need to balance electricity generation and demand at all times. In the Union of the Co-ordination of Transmission of Electricity (UCTE) this load-frequency control is maintained by procuring three reserve qualities differing in terms of activation and response speed [1]. Primary reserve (regulation) and secondary reserve (spinning reserve) are automatically called and must be provided within 30 s and 5 min, respectively. Tertiary reserve (spinning and non-spinning reserve) is called via rescheduling of generation and must be provided within 15 min. Such system balancing is usually in the responsibility of a transmission system operator (TSO) with an ex ante definition of the quantity of power systems reserve to be procured.

Due to requirements during merger control and threatened malpractice actions by the federal cartel office the TSO in Germany were obliged to procure the various types of power systems reserve by way of competitive tendering [2]. They decided to establish procurement auctions characterized by simultaneous tendering of multiple generation units. Thereby all generation units fulfilling defined requirements are allowed to bid. Any bid consists of the offered capacity and two prices. One price is for holding the capacity in reserve and the other for delivery in case of actual use. For the remuneration of the accepted bids the pay-as-bid settlement-rule is applied. This distinct market design may be summarized to be a multi-unit multi-part pay-as-bid procurement auction.

This is a working paper presenting preliminary research findings. You are advised to not cite this paper unless you first contact the author regarding possible amendments.

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In general, there are many other ways to design an auction for procuring power systems reserve. Consequently, an increasing amount of work can be found in the literature that deals with the design of such markets. One line of research develops algorithms for pricing power systems reserve in integrated market systems, cf. [3]–[5]. Some other work analyzes the efficiency of the settlement-rule and the question if pay-as-bid pricing is preferable to uniform pricing, cf. [6]–[8]. A further line of research considers the efficiency of the scoring-rule, given that the procurement auction for power systems reserve is based on multi-part bids, cf. [9]–[11]. This paper intends to add to this later line of research.

It has been mentioned that the procurement auctions for power systems reserve in Germany are based on submitting an offer that, next to the reserve capacity, consists of two parts: a capacity and an energy price bid. Thus the procurer needs to value the two price bids in order to decide on the offers to procure. This is based on a scoring-rule. Such rules have been developed for procuring several goods, for instance contracts for building highways [12] or weapons systems [13]. In these applications the scoring-rules value the elements of the offers with scores and the offer with the highest score is chosen.

The major question to be answered is: Which scoring-rule performs best for the procurer? Naturally this depends on the design of the auction. Here it is important to note that in Germany the costs for procuring power systems reserve are passed through to the consumers. Thus the scoring-rule should be designed for purpose of social efficiency. Additionally, for delivery in case of actual use the TSO is obliged to operate the reserve based on the merit order of the energy price bids. It is therefore crucial that the scoring-rule gives incentives for the bidders to reveal their respective variable generation costs in the energy price bids. Hence any strategic bidding behavior should be confined to the capacity price bids.

Such analysis on the scoring-rule can be found by Chao and Wilson [10] and Schummer and Vohra [11]. In both papers markets for power systems reserve are explicitly considered. However, the assumed market design differs considerably to the design of the German market. One important difference is the settlement-rule where, based on the Californian market, in both papers uniform pricing is assumed. This leads to a different scoring-rule performing best for the procurer as will be discussed in the remainder of this paper.

The paper is organized as follows: In Section II the design of the German reserve market is described and historic prices are analyzed. In Section III scoring-rules for procuring power systems reserve are discussed. In Section IV an exemplary application of the scoring-rules is presented. Finally, in Section V conclusions are drawn.

II. Power Systems Reserve

A. Load-Frequency Control

In order to secure power supply at all times and as electricity is nearly none storable, balancing of electricity systems is of crucial importance. Any imbalance of generation and demand will result in load-frequency changes that can lead to an extended breakdown of the network. Reasons for such imbalances are (i) imprecise forecasts of electricity demand or wind power generation and (ii) unforeseen events such as plant outages or network disturbances.

In Germany balancing the system follows the regulations set by the UCTE [1]. Thereby control actions are performed in successive steps, each with different characteristics and qualities, and all depending on each other:

- primary control starts within seconds;
- secondary control replaces primary after minutes;
- tertiary control frees secondary after rescheduling.

An imbalance of the system will cause all generators under primary control to respond within less than 30 s. Thereby the P-controllers of primary control automatically adjust the generation until the system balance is re-established. Then the system frequency remains at a quasi-steady-state value that differs from the system frequency set-point and changes the cross-border power exchanges. This automatically leads the PI-controllers of secondary control to take over within 5 min. In case of major imbalances secondary control may not be sufficient. In such cases tertiary control is activated. Tertiary control is called manually via rescheduling of generation and must be provided within 15 min.

Within a defined region system balancing is in the responsibility of one of four (more or less) independent TSO. As they do not own power plants due to unbundling they procure power systems reserve of approximately 7400 MW by way of competitive tendering.

B. Market Design

Competitive tendering started due to merger control requirements and threatened malpractice actions by the federal cartel office in the year 2001. It has been mentioned before that the design of an efficient scoring-rule depends greatly on the design of the considered procurement auction. Therefore the chosen tendering model in Germany will briefly be described below and is schematized in Fig. 1.

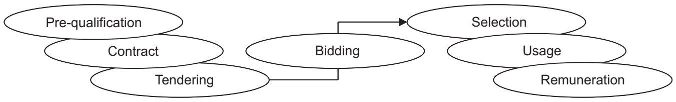

Fig. 1. Schematized tendering model.

1) Pre-qualification: Potential suppliers of power systems reserve can participate in a procedure under which the technical capabilities of the generation units are reviewed by the TSO (such as the accuracy of frequency measurement, power gradient, capability of load-following, energy availability factor, availability time ratio [14]). Following successful pre-qualification, the supplier is entitled to participate in the tendering procedures.

2) Contract: The commercial and administrative outline conditions are defined in a contract between the potential bidder and the TSO before the first bid can be submitted.

3) Tendering: Each TSO operates his own web-based market. These markets are one-sided procurement auctions as a market participant can either offer or ask for reserves, that are generators and the TSO respectively. The markets are multi-unit auctions as more than one reserve unit is auctioned at the same time. Thereby ex ante defined reserve capacities are procured that are price inelastic and known to the potential bidders. Furthermore, the markets for secondary and tertiary reserve are multi-part auctions reflecting that each offer is based on two prices, a capacity and an energy price. Primary reserve is a single-part auction as each offer is based on an energy price only. Each reserve quality is procured separately in positive and negative direction. While tendering for primary and secondary reserve takes place every six months, tendering for tertiary reserve is carried out daily.

4) Bidding: A bidder will determine his offers with respect to maximizing his expected profit. As there are four markets, the bidder needs to determine the offer prices and capacities for each market separately. For this several aspects need to be considered. One aspect is that each bidder must at least bid a minimal capacity and if a TSO accepts the offer at least this minimal capacity must be procured. Another aspect is that the markets are characterized by different offer periods lasting several hours and different times of market opening.

5) Selection: The bids are procured on the basis of economical and technical criteria. In general, the TSO see to it that network stability and operational security concerns are sufficiently taken into consideration. It is however often sufficient to assume that the respective reserve forms a homogenous product. This assumption is justified with the generation units fulfilling defined technical requirements as reviewed during the pre-qualification. Then non-discriminatory procuring of power systems reserve can be based on the price bids. As each offer consists of two prices the TSO applies a scoring-rule to value the offers and to decide which bids to procure. Currently this scoring-rule disregards the energy price bids and hence is based on the merit order of the capacity price bids only.

6) Usage: The procured reserves are activated based on the merit order of the energy price bids.

7) Remuneration: For remunerating the accepted bids the pay-as-bid settlement-rule is applied. Thus any accepted offer is paid with the respective bidding prices. It is worth to note that with pay-as-bid pricing, the bidders incentive is to bid as close to the a priori unknown clearing price as possible. Hence, all bidders may bid higher than marginal costs with rewards to those that can best guess the clearing price. In difference to uniform-pricing this means that for any bidder the decisions on the bidding prices will influence the expected profit.

C. Historic Prices

Based on these brief descriptions an analysis of the historic prices may indicate if the simple scoring-rule applied by the TSO leads to inefficient results. This analysis considers the market outcomes of procuring incremental tertiary reserve that is used to balance a shortage of supply. Thereby the workdays averaged capacity prices, the demanded and bided capacities and the maximal energy prices are studied. Of the four TSO in Germany the market of RWE Net AG [15] is considered (the price developments of the other markets exhibit similar characteristics). The data starting August 1, 2001 and ending March 31, 2004 is given in Fig. 2. For discussions on the statistics of the price developments cf. [16].

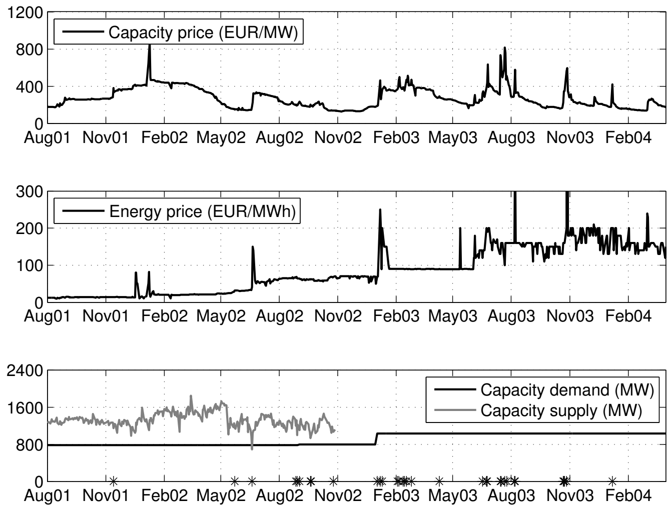

Fig. 2. Daily averaged capacity prices (top), maximal energy prices (middle) and daily averaged demanded and supplied capacities (bottom) on workdays; the asterix indicate product times with a shortage of supplied capacities.

With Fig. 2 can be seen that the reserve market prices exhibit the following characteristics:

- moderate volatility;
- frequent price spikes;
- regime shifts.

The price fluctuations on the reserve market are not as high as those commonly observed on electricity spot markets. One of the main reasons is that on the spot market (energy) demand depends on exogenous factors while (capacity) demand on the reserve market is fixed. Hence, a bidder on a reserve market hardly reacts to exogenous factors.

The RWE market starts after the spot market price is announced. The frequent price spikes can thus often be explained by jumps on the spot market [2] that sometimes also lead to a low liquidity on the reserve market, cf. Fig. 2 (bottom). This indicates that a higher fraction of generation units able to provide reserve and with comparatively high marginal costs, as for example gas turbines, have already been sold on the spot market (possibly due to a price jump there). Thereby may be noteworthy that the opportunity to submit an offer to the spot market is the major driver for a bidder to decide on the capacity price bid. In principle, price spikes are often followed by a regime shift to a higher price level. Thereby a tendency to remain on such a level can be observed.

Another important aspect to discuss is that over time the maximal energy prices show a tendency to increase while the daily averaged capacity prices remain at a constant or even decreasing level, cf. Fig. 2 (middle) and (top) respectively. The at least constant level of the capacity prices may be explained by an increasing price-competition. One reason is that the number of pre-qualified bidders has increased, for instance via pooling that allows small suppliers to participate in the auction. But one question remains: Why is the level of the energy prices unaffected by this development?

Before answering this question it may be useful to explain why this is at all important. The reason is that the costs for securing supply and here for procuring power systems reserve are passed through to the consumers. The costs for the capacity payment are directly passed through to the network charges and the energy prices are first remunerated by unbalanced generators but then passed through to the energy prices. Thus, a socially efficient market outcome is most desirable.

The answer to the question above may be the design of the scoring-rule used by the TSO to decide on the submitted offers. This scoring-rule leads to procure the offers regardless of the energy price bid. Considering the market design and thereby especially the pay-as-bid settlement-rule, this may lead the bidders to strategically decide on both price bids based on maximizing their expected profit. The decision on the capacity price reflects the expectation to be procured and the decision on the energy price reflects the expectation of delivery.

Following this discussion a bidder could try to optimize the position in the merit order of the accepted energy price bids. This may result to energy price bids higher than the respective variable generation costs as seen in Fig. 2 (middle). Note that such high energy price bids will not alter the procurement decision as long as the capacity price is competitive. Hence, the historic prices indicate that the simple scoring-rule used today may not be able to give incentives that the bidders reveal their respective variable generation costs in the energy price bids. This leads to think about possible alternatives.

III. Proposed Methodology

A. Problem Formulation

In the following methodologies are discussed that are based on the design of the German day-ahead markets for incremental tertiary reserve. Thereby a few assumptions are considered:

A1) The capacity to be procured by the TSO is constant, price inelastic and known to the potential bidders.

A2) A bidder must at least bid a minimal capacity. If the TSO accepts the bid at least this capacity must be procured.

A3) The product is homogeneous. The procurement decision is thus based on the two-part price bids only.

A4) The scoring-rule is not published. A bidder may derive the rule by analyzing historic procurement decisions.

A5) The publication of the procurement decisions consists of the bidding capacities and prices of all offers.

A6) All market actors are risk neutral. Each bidder is a price-taker but may bid strategically.

Procuring power systems reserve is driven by the objective to minimize the expected costs $\tilde{C}$. The major constraint is that the TSO procures offers $\mathrm{o}_j \in \mathbb{J}$ of the set $\mathbb{J} = \{\mathrm{o}_1, \mathrm{o}_2, \ldots, \mathrm{o}_J\}$ covering the reserve capacity demand $L^{\mathrm{D,max}}$. Thus implicitly high costs for an unserved demand are taken into account. An offer consists of a bidding capacity $L_j^{\mathrm{B}} \in \mathbb{R}_+$ and a two-part price bid. For the former holds $L_j^{\mathrm{B}} \geq L^{\mathrm{D,min}}$, i.e. the bidder must at least bid a defined minimal capacity. The later consists of a capacity price $p_j^{\mathrm{L}} \in \mathbb{R}_+$ and an energy price $p_j^{\mathrm{E}} \in \mathbb{R}_+$.

From all the offers in $\mathbb{J}$ the TSO procures a set $\mathbb{I}$ with $\mathbb{I} \subseteq \mathbb{J}$ and thus $I \leq J$ by minimizing the expected costs of procuring. Thereby a binary variable $z_j \in \{0,1\}$ is defined that takes the value $z_i = 1 \; \forall \; \mathrm{o}_i \in \mathbb{I}$ if an offer is accepted. Each offer can be procured with a capacity lower than the bidding capacity; for this accepted capacity holds $L^{\mathrm{D,min}} \leq L_i^{\mathrm{A}} \leq L_i^{\mathrm{B}}$.

The offers $\mathrm{o}_i \in \mathbb{I}$ are activated based on the merit order of the energy price bids. Thereby two aspects are noteworthy. First, the TSO can activate a fraction of the accepted capacity of an offer. Second, the TSO may activate a single offer more than once within the period of holding the capacity in reserve. This leads defining the energy demand $E_i^{\mathrm{D}} \in \mathbb{R}_+$ covered by an accepted offer to be the product of the accepted capacity L_i^{A} and its computed duration of actual use $h_i^{\mathrm{D}} \in \mathbb{R}_+$.

Here the remuneration is based on the pay-as-bid settlement-rule. Hence the procurement costs C for the TSO can ex post be calculated as the sum of the products of the specific costs c_i and the accepted capacities of all procured offers:

$$C = \sum_{i=1}^{I} L_i^{\mathrm{A}} \, c_i \quad \text{with} \quad c_i = p_i^{\mathrm{L}} + h_i^{\mathrm{D}} \, p_i^{\mathrm{E}} \qquad (1)$$

While at the time of remuneration the accepted capacity and the energy demand of each offer is known, the later is unknown at the time of selection. Hence, if a scoring-rule shall consider both price bids of the submitted offers, an adequate description of the expected activation of an accepted offer is needed. It is however first necessary to formalize the bidding problem to be able to analyze the scoring-rule's efficiency later on.

The method is based on the assumption that the market price can be seen as a stochastic variable $p^{\mathrm{M}} \in \mathbb{R}_+$ following the density function $f^{\mathrm{M}}(p) : \mathbb{R}_+ \mapsto \mathbb{R}_+$ with the probability distribution $F^{\mathrm{M}}(p) : \mathbb{R}_+ \mapsto [0,1]$. This leads to calculate a probability of acceptance summarizing the behavior of the competing bidders:

$$\mathrm{P}^{\mathrm{A}}(p^{\mathrm{M}} > p^{\mathrm{B}}) = 1 - F^{\mathrm{M}}(p^{\mathrm{B}}) = 1 - \int_{-\infty}^{p^{\mathrm{B}}} f^{\mathrm{M}}(p) \, dp \qquad (2)$$

Thereby $p^{\mathrm{B}} \in \mathbb{R}_+$ gives the bidding price. By further considering the bidding capacity L^{B} and the bidding costs $c^{\mathrm{B}} \in \mathbb{R}_+$ the expected profit $\tilde{\Pi}$ can be calculated:

$$\max_{\{p^{\mathrm{B}}\}} \quad \tilde{\Pi} = \mathrm{P}^{\mathrm{A}}(p^{\mathrm{M}} > p^{\mathrm{B}}) \, L^{\mathrm{B}} \left(p^{\mathrm{B}} - c^{\mathrm{B}}\right) \qquad (3)$$

Note that this is a simple method with the bidder assumed to be a price-taker. This results in a negligible bidding capacity and no influence on the market price. Especially the later is a strong limitation due to the pay-as-bid settlement-rule. However, the method is sufficient for the application at hand. For a detailed treatment solving these limitations cf. [17].

B. Naive Approach

Initially the validity of a constant expected duration of using an offer $\tilde{h}^{\mathrm{D}} \in \mathbb{R}_+$ may be assumed. The expected duration of using an offer may thus be independent of an offer's bidding capacity and energy price. This leads to formalize:

$$\tilde{c}_j = p_j^{\mathrm{L}} + \tilde{h}^{\mathrm{D}} \, p_j^{\mathrm{E}} \quad \text{with} \quad \tilde{h}^{\mathrm{D}} = \tilde{h}_j^{\mathrm{D}} = \mathrm{const} \; \forall \; \mathrm{o}_j \in \mathbb{J} \qquad (4)$$

With this scoring-rule the expected costs $\tilde{c}_j$ of each offer are calculated based on a given expected duration of using an offer. For the later holds $0 \leq \tilde{h}^{\mathrm{D}} \leq h^{\mathrm{D,max}}$. Thereby $h^{\mathrm{D,max}}$ gives the maximal period of holding the accepted capacity in reserve. The TSO may derive the expected duration of using an offer by analyzing the historic use of procured reserves.

Given the problem formulation the accepted capacity need to be greater than a defined minimal capacity. Thus a binary variable is needed that gives the status of accepting or rejecting a submitted offer. This leads to the following linear mixed-integer objective function:

$$\begin{aligned} &\min_{\{z_j,\, L_j^{\mathrm{A}}\}} \quad \tilde{C} = \sum_{j=1}^{J} L_j^{\mathrm{A}} \left(p_j^{\mathrm{L}} + \tilde{h}^{\mathrm{D}} \, p_j^{\mathrm{E}}\right) \\ &\text{s.t.} \quad \textstyle\sum_{j=1}^{J} L_j^{\mathrm{A}} = L^{\mathrm{D,max}} \qquad (5) \\ &\qquad z_j \, L^{\mathrm{D,min}} \leq L_j^{\mathrm{A}} \leq z_j \, L_j^{\mathrm{B}} \\ &\qquad z_j \in \{0,1\} \end{aligned}$$

This approach is easy to apply and allows to set up a merit order of the expected costs of all submitted offers. The offers with the lowest expected costs are procured until the reserve capacity demand is covered. It is evident that in the limiting case of $\tilde{h}^{\mathrm{D}} = 0$ this approach reflects the scoring-rule applied in today's practice by the TSO in Germany.

One aspect to question is whether the initial assumption of a constant expected duration of using an offer is valid. Given that activating the accepted offers follows the merit order of the energy price bids, it can be expected that the probability of activation decreases with an increasing energy price. Hence, a constant expected duration of using an offer does not seem to adequately represent the reality.

Furthermore, if today's scoring-rule is based on the capacity price bid only, this leads to conclude that the TSO expects the reserve never to be activated. Then the question arises: Why at all procuring reserves? Hence, the procurement a priori results in an expected duration of using to be greater than zero.

In spite of these limitations this simple scoring-rule is used in today's daily operation of the TSO in Germany. It is hence worth to go further into the question if this scoring-rule is efficient and leads a bidder to reveal the variable generation costs in the energy price bid.

Proposition 1: If the scoring-rule in (5) is applied without considering any expected reserve energy demand for selecting the offers then a bidder's optimal energy price bid will be higher than the respective variable generation costs $c^{\mathrm{E}} \in \mathbb{R}_+$.

Proof: Following the discussions above the bidder may consider a positive and monotonic decreasing function of the expected duration of using an offer. This function may depend on the bidder's energy price bid and the expectation on the merit order of the competitors accepted energy price bids. Here this may be expressed by $\tilde{h}^{\mathrm{B}}(p^{\mathrm{E}}) : \mathbb{R}_+ \mapsto \mathbb{R}_+$. Note that with this function the bidder uses more information on the expected duration of using an offer than the TSO with the scoring-rule. With (2) and (3) the bidder's expected profit can then be calculated by:

$$\tilde{\Pi} = \left(1 - F^{\mathrm{M}}(p^{\mathrm{L}})\right) L^{\mathrm{B}} \left(p^{\mathrm{L}} - c^{\mathrm{L}} + \tilde{h}^{\mathrm{B}}(p^{\mathrm{E}}) \left(p^{\mathrm{E}} - c^{\mathrm{E}}\right)\right) \quad (6)$$

The optimal energy price bid can be derived by setting the first derivative of (6) at p^{E} equal to zero:

$$\left(1 - F^{\mathrm{M}}(p^{\mathrm{L}})\right) L^{\mathrm{B}} \left(\tilde{h}^{\mathrm{B}}(p^{\mathrm{E}}) + \frac{\partial \tilde{h}^{\mathrm{B}}(p^{\mathrm{E}})}{\partial p^{\mathrm{E}}} \left(p^{\mathrm{E}} - c^{\mathrm{E}}\right)\right) = 0 \quad (7)$$

This yields the optimal energy price bid to follow:

$$p^{\mathrm{E}} = c^{\mathrm{E}} - \frac{\tilde{h}^{\mathrm{B}}(p^{\mathrm{E}})}{\frac{\partial}{\partial p^{\mathrm{E}}} \tilde{h}^{\mathrm{B}}(p^{\mathrm{E}})} \quad (8)$$

As the bidder is assumed to consider a function of the expected duration of using a bid that is positive, $\tilde{h}^{\mathrm{B}}(p^{\mathrm{E}}) > 0$, and monotonic decreasing, $\partial \tilde{h}^{\mathrm{B}}(p^{\mathrm{E}})/\partial p^{\mathrm{E}} < 0$, the energy price bid will exceed the variable generation costs, $p^{\mathrm{E}} > c^{\mathrm{E}}$. ■

This allows to conclude that the scoring-rule applied by the TSO in Germany does not lead a bidder to reveal the variable generation costs in the energy price bid. It is therefore not surprising that an increasing trend in the historic energy price bids can be seen. The scoring-rule is thus not efficient.

C. Duration Curve Approach

Following the discussion above the application of the naive approach by the TSO in Germany can neither be explained by an appropriate representation of the ex ante unknown reserve energy demand nor by a minimization of strategic incentives for the bidders. The choice for this scoring-rule may thus either be due to its simplicity or to a lack of knowledge regarding the induced strategic incentives. This leads to think about an approach that allows to procure efficiently by an improved representation of the expected reserve energy demand.

For that purpose the inverse of a reserve power duration curve can be used. This follows an approach formulated by Bushnell and Oren [9]. Thereby the authors propose to value submitted offers based on the evaluation of a load duration curve (this relates to the theory of peak-load pricing [18]). It is thereby presumed that an ex ante unknown variable can be described stochastically.

A duration curve is an ordered representation of an observed variable as a function of the cumulative number of observations in a given time period. The inverse of a duration curve is thus a representation of the duration as a function of the observed variable. If this duration is normalized to one the inverse of a duration curve $\breve{F}^{\mathrm{D}}(x) : \mathbb{R}_+ \mapsto [0,1]$ can be interpreted as the expected fraction of the duration at which the observed variable χ is greater than a value x.

The normalized inverse of a duration curve is thus nothing else than the inverse of a probability distribution based on historic data and given by $F^{\mathrm{D}}(x) : \mathbb{R}_+ \mapsto [0,1]$. Formally it holds $\breve{F}^{\mathrm{D}}(x) = \mathrm{P}^{\mathrm{D}}(\chi > x) = 1 - F^{\mathrm{D}}(x)$, i.e. following (2) the normalized inverse of a duration curve can be calculated as the integral of a density function $f^{\mathrm{D}}(x) : \mathbb{R}_+ \mapsto \mathbb{R}_+$.

Here the observed variable χ corresponds to the activated reserve capacity L^{D} caused by unforeseen events. This leads to characterize the deducible normalized inverse duration curve as the inverse reserve capacity duration curve or simply the probability of activation. A multiplication with the maximal period of holding an accepted capacity in reserve, $h^{\mathrm{D,max}}$, gives the expected duration of activation as a function of the reserve capacity. This is visualized in Fig. 3 (left).

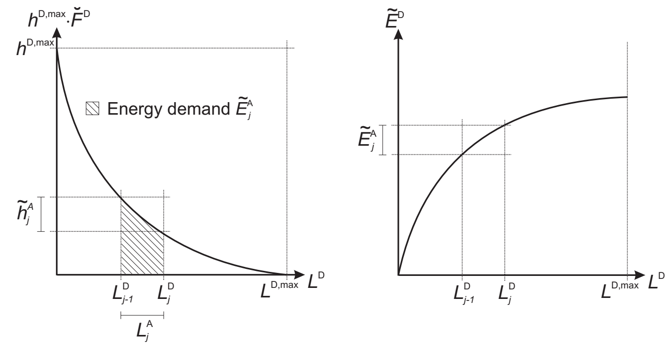

Fig. 3. Visualization of the duration curve approach with the inverse reserve capacity duration curve (left) and the expected reserve energy demand (right).

The integral under this curve can be understand to form the expected reserve energy demand, visualized in Fig. 3 (right). With the reserve capacity L_j^{D} defined to be the sum of the accepted capacities L_i^{A} of all $j \le I$ offers and thus be given by $L_j^{\mathrm{D}} = \sum_{i=1}^{j} L_i^{\mathrm{A}}$, the integral can be calculated following:

$$\tilde{E}^{\mathrm{D}}(L_j^{\mathrm{D}}) = h^{\mathrm{D,max}} \int_0^{L_j^{\mathrm{D}}} \breve{F}^{\mathrm{D}}(L^{\mathrm{D}})\, dL^{\mathrm{D}} \quad (9)$$

In order to formulate the objective function minimizing the procurement costs $\tilde{C}$ and by considering the proposed duration curve approach as a scoring-rule, it is first necessary to have the offers in a merit order of the energy price bids. This allows to allocate a decreasing expected duration of activating an offer with an increasing energy price bid. The allocated expected reserve energy demand of an offer can then be calculated using (9) by the difference $\tilde{E}^{\mathrm{D}}(L_j^{\mathrm{D}}) - \tilde{E}^{\mathrm{D}}(L_{j-1}^{\mathrm{D}})$. This leads to the following non-linear mixed-integer objective function:

$$\min_{\{z_j,\, L_j^{\mathrm{A}}\}} \quad \tilde{C} = \sum_{j=1}^{J} L_j^{\mathrm{A}}\, p_j^{\mathrm{L}} + \left[\tilde{E}^{\mathrm{D}}(L_j^{\mathrm{D}}) - \tilde{E}^{\mathrm{D}}(L_{j-1}^{\mathrm{D}})\right] p_j^{\mathrm{E}}$$

$$\begin{aligned} \text{s.t.} \quad & \textstyle\sum_{j=1}^{J} L_j^{\mathrm{A}} = L^{\mathrm{D,max}} \\ & z_j\, L^{\mathrm{D,min}} \le L_j^{\mathrm{A}} \le z_j\, L_j^{\mathrm{B}} \\ & z_j \in \{0,1\} \\ & \textstyle\sum_{k=1}^{j} L_k^{\mathrm{A}} = L_j^{\mathrm{D}} \end{aligned} \quad (10)$$

The scoring-rule given in (10) can in principle be compared to the approaches proposed by Chao and Wilson [10] and Schummer and Vohra [11]. However, in both papers several simplifications prevent their application given the market design in Germany. The former consider a competitive market, the uniform settlement-rule, a known probability of activating an accepted offer, no defined capacity demand and no minimal bidding capacity. The later do not consider a competitive market but some further simplifications, namely a known and discrete probability of activating an offer and that if an accepted capacity is activated it has to deliver over the whole duration of holding the capacity available.

Clearly this approach is not as easy to apply as the naive scoring-rule. However, the solvability greatly depends on the assumptions regarding the probability distribution representing the inverse reserve capacity duration curve. This is due to today's powerful solvers requiring the derivatives of the objective function to be given analytically. But before analyzing the applicability of the scoring-rule it might be useful to discuss, if this approach can be seen to be efficient.

Proposition 2: If the scoring-rule in (10) is applied, then in contrast to the application of the scoring-rule in (5) a bidder's optimal energy price bid will not necessarily be higher than the respective variable generation costs.

Proof: The bidder may consider a positive and monotonic decreasing function of the expected duration of using an offer. This function may depend on the bidder's energy price bid and the expectation on the merit order of the competitors accepted energy price bids and be expressed by $\tilde{h}^{\mathrm{B}}(p^{\mathrm{E}}) : \mathbb{R}_+ \mapsto \mathbb{R}_+$. Note that this function does not necessarily equal the scoring-rule applied by the TSO as the later is assumed not to be published. The bidder thus cannot have a better knowledge of the expected duration of using an offer than the TSO. Hence, the bidder faces an additional uncertainty. With (2) and (3) the bidder's expected profit can then be calculated by:

$$\begin{aligned}\tilde{\Pi} = &\left(1 - F^{\mathrm{M}}(p^{\mathrm{L}} + \tilde{h}^{\mathrm{B}}(p^{\mathrm{E}})p^{\mathrm{E}})\right) L^{\mathrm{B}} \\ &\times \left(p^{\mathrm{L}} - c^{\mathrm{L}} + \tilde{h}^{\mathrm{B}}(p^{\mathrm{E}})\left(p^{\mathrm{E}} - c^{\mathrm{E}}\right)\right)\end{aligned} \tag{11}$$

The optimal energy price bid can then be derived by setting the first derivative of (11) at p^{E} equal to zero:

$$\begin{aligned}&\left(1 - F^{\mathrm{M}}(p^{\mathrm{L}} + \tilde{h}^{\mathrm{B}}(p^{\mathrm{E}})p^{\mathrm{E}})\right) \\ &\times L^{\mathrm{B}}\left(\tilde{h}^{\mathrm{B}}(p^{\mathrm{E}}) + \frac{\partial}{\partial p^{\mathrm{E}}}\tilde{h}^{\mathrm{B}}(p^{\mathrm{E}})\left(p^{\mathrm{E}} - c^{\mathrm{E}}\right)\right) \\ &- \frac{\partial}{\partial p^{\mathrm{E}}} F^{\mathrm{M}}(p^{\mathrm{L}} + \tilde{h}^{\mathrm{B}}(p^{\mathrm{E}})p^{\mathrm{E}}) \\ &\times L^{\mathrm{B}}\left(p^{\mathrm{L}} - c^{\mathrm{L}} + \tilde{h}^{\mathrm{B}}(p^{\mathrm{E}})\left(p^{\mathrm{E}} - c^{\mathrm{E}}\right)\right) = 0\end{aligned} \tag{12}$$

With the abbreviation $F^{\mathrm{M}} = F^{\mathrm{M}}(p^{\mathrm{L}} + \tilde{h}^{\mathrm{B}}(p^{\mathrm{E}})p^{\mathrm{E}})$ this yields the optimal energy price bid to follow:

$$p^{\mathrm{E}} = c^{\mathrm{E}} + \frac{\left(p^{\mathrm{L}} - c^{\mathrm{L}}\right)\frac{\partial}{\partial p^{\mathrm{E}}}F^{\mathrm{M}} - \left(1 - F^{\mathrm{M}}\right)\tilde{h}^{\mathrm{B}}(p^{\mathrm{E}})}{\left(1 - F^{\mathrm{M}}\right)\frac{\partial}{\partial p^{\mathrm{E}}}\tilde{h}^{\mathrm{B}}(p^{\mathrm{E}}) - \tilde{h}^{\mathrm{B}}(p^{\mathrm{E}})\frac{\partial}{\partial p^{\mathrm{E}}}F^{\mathrm{M}}} \tag{13}$$

As the bidder is assumed to consider a function of the expected duration of using a bid that is positive, $\tilde{h}^{\mathrm{B}}(p^{\mathrm{E}}) > 0$, and monotonic decreasing, $\partial\tilde{h}^{\mathrm{B}}(p^{\mathrm{E}})/\partial p^{\mathrm{E}} < 0$, and as the distribution and its derivative are positive, $F^{\mathrm{M}}(p^{\mathrm{L}} + \tilde{h}^{\mathrm{B}}(p^{\mathrm{E}})p^{\mathrm{E}}) > 0$ and $\partial F^{\mathrm{M}}(p^{\mathrm{L}} + \tilde{h}^{\mathrm{B}}(p^{\mathrm{E}})p^{\mathrm{E}})/\partial p^{\mathrm{E}} > 0$, the denominator in (13) will always be negative. As furthermore the energy price bid will never be lower than the variable generation costs, $p^{\mathrm{E}} \geq c^{\mathrm{E}}$, for the optimal capacity price bid ($p^{\mathrm{L}} \geq c^{\mathrm{L}}$) follows:

$$p^{\mathrm{L}} \leq c^{\mathrm{L}} + \frac{\left(1 - F^{\mathrm{M}}(p^{\mathrm{L}} + \tilde{h}^{\mathrm{B}}(p^{\mathrm{E}})p^{\mathrm{E}})\right)\tilde{h}^{\mathrm{B}}(p^{\mathrm{E}})}{\frac{\partial}{\partial p^{\mathrm{E}}}F^{\mathrm{M}}(p^{\mathrm{L}} + \tilde{h}^{\mathrm{B}}(p^{\mathrm{E}})p^{\mathrm{E}})} \tag{14}$$

If the equality holds, the bidder's expected profit is maximized and the energy price bid reveals the respective variable generation costs. ■

Note that if the bidder would ex ante know the scoring-rule applied by the TSO and the merit order of the energy price bids of the competitors, the bidder would be indifferent on the optimal decision on both price bids. Under such an assumption (and given that the bidder is risk neutral) the energy price bids may not reveal the respective variable generation costs.

Even though this situation is most unlikely to happen in a real world application, it would still lead to more efficient market outcomes as the expected reserve energy demand is much better represented. However, due to the additional uncertainty on the scoring-rule the bidder will most likely decide to set the energy price bid equal to the variable generation costs. This reduces the bidder's uncertainty upon the activation of his offer and can lead to an optimal expected profit.

This allows to conclude that the scoring-rule based on the inverse reserve power duration curve leads a bidder to reveal the variable generation costs in the energy price bid. It can be presumed that the application of this scoring-rule by the TSO in Germany may lead the energy price bids to decrease and the scoring-rule to yield efficient market outcomes.

IV. Exemplary Application

A. Bidding Data

The applicability of the scoring-rules is shown based on data of the RWE market for procuring incremental tertiary reserve in Germany [15]. One aspect to discuss is the density function $f^{\mathrm{D}}(x)$ that defines the scoring-rule following the duration curve approach in (10). Here the validity of an exponential distribution with rate parameter $b = 10.32$ is assumed:

$$f^{\mathrm{D}}(x) = \begin{cases} 0 & \text{, if } x \leq 0 \\ \frac{1}{b}\exp\left(-\frac{x}{b}\right) & \text{, if } x > 0 \end{cases} \tag{15}$$

This function has been derived using publicly available data on historic procuring results and reserve energy demand. Thus this function does not necessarily represent the reality. For instance, in a real world application other distributions may be better able to represent the actual expectations. One possibility could be a truncated Gaussian-mixture distribution that offers a higher degree-of-freedom to fit the parameters and better representing the actual data.

Here the exponential distribution is chosen as this allows to easily derive the first and second derivatives of the objective function in (10). They are needed for solving the mixed-integer non-linear problem using Matlab® and the `minlpBB` function of the MINLP toolbox of Tomlab. This allows to find the solution within 5 s on a Intel® Pentium® M Prozessor 1 GHz.

For the application some additional data characterizing the RWE market are needed, namely the maximal period of holding a capacity in reserve of $h^{\mathrm{D,max}} = 4\,\mathrm{hrs}$, the minimal bidding capacity of $L^{\mathrm{D,min}} = 30\,\mathrm{MW}$ and the reserve capacity demand of $L^{\mathrm{D,max}} = 750\,\mathrm{MW}$. The actual bidding data and the procuring results based on the scoring-rule following the naive approach, SR_1, in (5) and the duration curve approach, SR_2, in (10) are given in Table I.

TABLE I

BIDDING DATA AND COMPARATIVE PROCURING RESULTS.

o_j	L^{B}_j (MW)	p^{L}_j (€/MW)	p^{E}_j (€/MWh)	$\mathrm{SR}_1{}^*$: L^{A}_j (MW)	$\mathrm{SR}_2{}^*$: L^{A}_j (MW)
1	60	11.90	138.00	60	60
2	100	12.00	134.00	100	100
3	100	12.01	133.10	100	100
4	100	12.02	134.20	100	100
5	100	12.03	133.30	100	100
6	200	12.09	134.00	200	197
7	100	12.10	280.00	90	30
8	150	12.19	133.00	0	0
9	100	12.30	110.00	0	33
10	100	12.50	125.00	0	0
11	30	53.80	21.90	0	30
12	50	59.00	25.00	0	0

* Comparison of procuring results with selected scoring-rules (SR):
– SR_1: $\tilde{h}^{\mathrm{D}} = 0\,\mathrm{hrs}$
– SR_2: Exponential distribution according (15) with $b = 10.32$

Here the application of SR_1 leads to procure by neglecting the energy price bids. Thus offers 1 to 7 having the lowest capacity price bids are procured. The application of SR_2 leads to considerably different procurement results. Here offer 6 and 7 are partly procured and the remaining capacity demand is covered by also partly procuring offer 9 and 11. Those are chosen due to the higher capacity and lower energy price bids.

B. Cost Isoquants

Following these procurement results and the theoretic discussions above, it can be concluded that the choice of the scoring-rule will yield to different incentives for a strategic bidding behavior. A possible illustration is to plot the accepted and rejected offers in a diagram of the capacity over the energy price bids. This allows drawing the cost isoquants of the submitted offers. A cost isoquant reflects the attributable expected duration of activating an offer by its slope. It indicates that any offer on such a straight line is cost equivalent to the procurer. Thus any additional offer will only be procured if it is located left or below such a cost isoquant.

In Fig. 4 (top) the cost isoquants applying SR_1 are given. It can be seen that offer 7 with its high energy price bid forms the marginal one. Thus any additional offer with a slightly lower capacity price bid would have been accepted; regardless of the energy price bid. Note that such an offer does not need to maximize the bidder's expected profit. This depends on the bidder's expectation on its actual use.

In Fig. 4 (bottom) the cost isoquants applying SR_2 are given. It can be seen that next to offer 7 also offers 9 and 11 with high capacity but low energy price bids form the marginal ones. Still the majority of the offers is procured by valuing the energy price bid with an expected duration of activation near zero, but those slight discrepancies lead to confine any strategic bidding behavior to the capacity price bids.

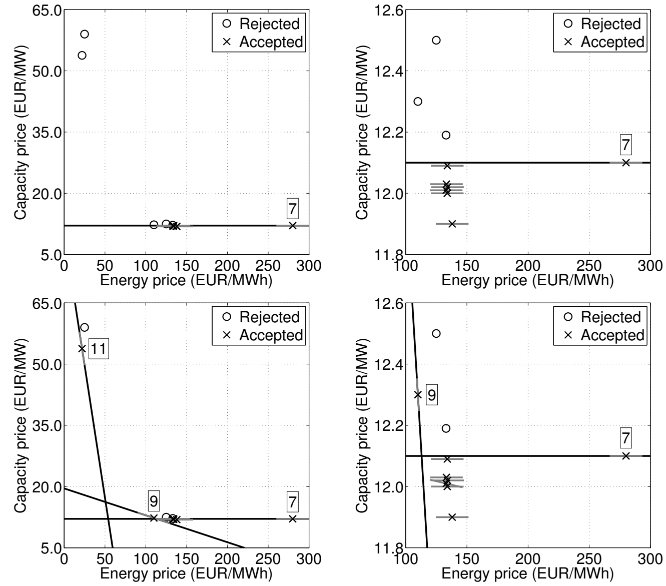

Fig. 4. Cost isoquants applying the naive approach (top) and the duration curve approach (bottom); the figures on the right hand side give a scaled representation of the figures on the left hand side only.

C. Expected Costs

Another interesting aspect applying an inadequate scoring-rule are the resulting expected procurement costs. Those are given for the considered bidding data in Table II.

TABLE II

FINANCIAL CONSEQUENCES OF AN INADEQUATE SCORING-RULE.

	Capacity costs	Expected energy costs		Expected total costs	
		resp. SR	SR_2	resp. SR	SR_2
	(€)	(€)	(€)	(€)	(€)
SR_1	9027.00	0.00	5494.37	9027.00	14521.37
SR_2	10284.59	1104.91	1104.91	11389.51	11389.51

First the capacity costs may be compared. Those can simply be calculated following $C^{\mathrm{L}} = \sum_{i=1}^{I} L^{\mathrm{A}}_i\, p^{\mathrm{L}}_i$. It is evident that the capacity costs are considerably lower applying SR_1. In case of applying SR_2 also offers with higher capacity price bids but comparatively lower energy price bids are considered.

Second the expected energy costs may be compared. Those can be calculated following $\tilde{C}^{\mathrm{E}} = \sum_{i=1}^{I} L^{\mathrm{A}}_i\, \tilde{h}^{\mathrm{D}}_i\, p^{\mathrm{L}}_i$ with the expected duration of activating an offer calculated by dividing the allocated expected reserve energy demand of an offer with the respective accepted capacity. If the expected energy costs are calculated using the expectation of the respective scoring-rule, then those costs are zero in case of SR_1 and thus much lower compared to SR_2.

However, this does not allow a reasonable comparison as the actual reserve energy demand is not negligible as assumed using SR_1. Here in both cases the expected reserve energy demand represented with SR_2 need to be considered as the later is assumed to represent the actual expectation. This leads to much higher expected energy costs applying SR_1 and following $\tilde{C} = C^{\mathrm{L}} + \tilde{C}^{\mathrm{E}}$ to nearly 28 % higher expected total costs compared to SR_2.

V. Conclusions

This paper contributes to the discussion on designing an efficient multi-part procurement auction for power systems reserve. Thereby the design of the markets operated by the TSO in Germany is considered. For paying the accepted bids the pay-as-bid settlement-rule is applied and each offer consists of a reserve capacity and two prices. The capacity price is for holding the offered capacity in reserve and the energy price is for delivery in case of actual use. Thus the procurer needs to value the two price bids in order to decide on the offers to procure. This can be based on a scoring-rule.

As the costs for procuring power systems reserve are generally passed through to the consumers and the activation of the reserves is principally based on the merit order of the energy price bids it is crucial that such a scoring-rule leads to an efficient market outcome. Thus the scoring-rule should give incentives for the bidders to reveal their respective variable generation costs in the energy price bids. The analysis of historic prices led to conclude that this cannot be achieved applying the simple scoring-rule neglecting the energy price bids as used today by the TSO in Germany.

Hence, in this paper the application of a scoring-rule is proposed that adequately reflects the ex ante unknown actual use of the procured reserve. With a time series of the historic reserve energy demand it is possible to derive a stochastic description. It is shown that this leads to a reserve power duration curve and yields to the derivation of the expected actual use of each offer with respect to the energy price bid. This finally results in a stochastic non-linear mixed-integer objective function for procuring power systems reserve by minimizing the expected costs.

It is shown that the duration curve approach can confine any strategic bidding behavior to the capacity price bids. This allows to conclude that this approach can help to guarantee that the procurer is able to maintain the current security levels at minimized expected costs. The approach can furthermore lead to an increased consumer surplus (from the procurer's perspective) that may then be passed through to the end consumers. Further improvements may be possible if the liquidity on the reserve markets is increased. One attractive solution in Germany may be to procure reserves before the spot market starts within a single auction.

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